TIME-VARYING ESTIMATES FOR THE NATURAL RATE OF UNEMPLOYMENT AND THE PHILLIPS CURVE IN THE US USING THE KALMAN FILTER

1. Introduction

One of the central issues in macroeconomics is whether there is any trade off between inflation and unemployment rate, the well-known Phillips (1958) curve. For conducting an efficient monetary policy the relationship between inflation and unemployment rate is of paramount importance since monetary changes affect both, and policymakers are often left with a monetary-policy conflict in pursuing the goals of low inflation and low unemployment. Relevant for the Phillips curve is the “natural rate of unemployment”, a term introduced by Friedman (1968). This concept was also developed in Phelps (1967, 1968). Later, there arose in the 1970s an approximately equivalent concept, known as the NAIRU (non-accelerating inflation rate of unemployment). If the unemployment rate is at the NAIRU level, inflation should equal expected inflation and there is no reason for expected inflation to change. However, if the unemployment rate is, for example, lower than the NAIRU level, then inflation is likely to be higher than expected according to the Phillips curve. That leads to a corresponding increase in expected inflation, so if the unemployment rate is sustained at its low level the following period, inflation needs to be even higher, higher than the new expected inflation level. In this way inflation must continue increasing to maintain unemployment below the NAIRU. A similar logic leads to inflation decreasing when unemployment is maintained above the NAIRU. Theoretically, the central bank cannot make the actual unemployment rate diverge from the NAIRU in the long run, but it can retain a constant rate of inflation if it is successful in keeping the actual unemployment rate equal to the NAIRU\textsuperscript{1}.

\textsuperscript{1} The NAIRU concept does have its critics, however. See for example Staiger, Stock and Watson (1997) and Galbraith (1997).
The NAIRU has often been estimated as a constant value in the past. However, there has been significant discussion about a falling NAIRU in the 1990s. In general, with the structure of the economy changing over time, there is no reason to expect an unchanging NAIRU. Some researchers have presented time-varying estimates for the US NAIRU, most notably Gordon (1997), King, Stock, and Watson (1995), Staiger, Stock, and Watson (1997), and Ball and Mankiw (2002). In these studies, the NAIRU is time varying, but the sensitivity of inflation to deviations of unemployment from the NAIRU is not. In this paper we present Kalman-filter estimates in which both this latter sensitivity measure and the NAIRU are time-varying.

This paper is organized as follows. Section 2 describes the Phillips curve and the NAIRU, and previous empirical studies on the subject. Section 3 presents the empirical methodology of the current paper. The estimation results are provided in Section 4. Conclusions are provided in the last section.

2. The Phillips Curve, NAIRU, and Previous Empirical Studies

The original Phillips curve was simply an empirical finding by Phillips (1958) that there was a negative relationship between wage inflation and wage rates during 1861-1957 for the UK, with zero wage inflation being associated with about a 6% unemployment rate. Lipsey’s (1960) explanation for this observed negative relationship is that lower unemployment is associated with higher excess labor demand, which is in turn associated with higher wage inflation. Later, negative relationships were found between price inflation and unemployment, to which the term “Phillips curve” was also attached. Theoretical justification for such a Phillips curve arises immediately if one accepts that an increase (decrease) in aggregate demand leads to increases (decreases) in employment and inflation in the short run. A linear statistical form of such a Phillips curve is given by

$$\pi_t = c - \alpha u_t + \varepsilon_t,$$

where $\pi_t$ signifies the inflation rate, $u_t$ is the unemployment rate, $\varepsilon_t$ is an error term which includes supply shocks, and $c$ and $\alpha > 0$ are parameters to be estimated.

US data prior to the early 1970s suggested to some economists that the relationship in (1) may be unchanging over time, but later data made that proposition difficult to support. Friedman (1968) and
Phelps (1967, 1968) had already called into question the theoretical validity of a long-run trade off between unemployment and inflation. According to their view such trade off can only exist if wage setters systematically mispredicted inflation. This misprediction of inflation will not prevail forever as the rational agents are going to improve their expectations. If the government attempts to sustain low unemployment by increasing inflation the trade-off will ultimately disappear as expected inflation increases. Friedman and Phelps argued that unemployment cannot be sustained below a certain level, which they called the natural rate of unemployment. A short-run trade-off as shown in (1) may continue exist, however, with the curve shifting over time due to changes in expected inflation.

More modern renditions of the Phillips curve include expected inflation, the simplest being

$$\pi_t = \pi_t^e - \alpha (u_t - u^*) + \epsilon_t$$

(2)

where $\pi_t^e$ is expected inflation and $u^*$ is the “natural rate of unemployment” or the NAIRU, the “non-accelerating inflation rate of unemployment”. If $u_t = u^*$, then inflation will equal its expected level plus some white noise error. Only when $u_t = u^*$ does inflation have tendency to be neither higher nor lower than expected. Low unemployment, in the sense of $u_t < u^*$, results in higher nominal wages, which causes firms to increase their prices. If unemployment is maintained at its low level, then in response to the resulting increase in prices the employees demand higher wages. This in turn results in higher prices, and higher wages and so on. The end result of this mechanism is increasing wage and price inflation, a “wage-price spiral”. The unemployment rate can be kept down below $u^*$ if the government is ready to accept increasing inflation rates. This option is not available in the long-run, however.

If we reorganize (2) and let $c = \alpha u^*$, the relationship to equation (1) becomes clearer, as shown in the equation below:

$$\pi_t = \pi_t^e - c - \alpha u_t + \epsilon_t$$

(3)

Compared to equation (1), we have simply added expected inflation as explaining current inflation. Unfortunately, we do not have data on expected inflation. However, we can, as is often done

---

2 As noted by Ball and Mankiw (2002), treating $u^*$ as the NAIRU is reasonable if rational expectations with respect to inflation are reasonably approximated by adaptive expectations. The reasonableness of such an approximation is discussed later.
empirically in estimating the NAIRU, use the adaptive expectations formulation $\pi_t^e = \pi_{t-1}$. This seems reasonable, as inflation appears to have approximately followed a random walk during the past four decades (Barsky, 1987; Ball, 2000; Ball and Mankiw (2002)). By replacing $\pi_t^e$ with $\pi_{t-1}$ in equation (3) and subtracting $\pi_{t-1}$ from both sides, we get

$$\pi_t - \pi_{t-1} = c - \alpha u_t + \varepsilon_t$$

(4)

from which we can find estimates for $c$ and $\alpha$, $\hat{c}$ and $\hat{\alpha}$ respectively. With these estimates we can determine an estimate for the NAIRU:

$$\hat{u}^* = \frac{\hat{c}}{\hat{\alpha}}$$

(5)

Estimation of equation (4) through ordinary least squares and calculating the NAIRU through equation (5) has been a common route to calculating the NAIRU. Using US annual data for 1970-1995, Blanchard and Katz (1997) for example estimated equation (4) to be

$$\pi_t - \pi_{t-1} = 6.64 - 1.02 u_t + \varepsilon_t$$

(6)

implying a NAIRU of $6.64/1.02 = 6.5$ percent. Using US annual data for 1960 to 2000, Ball and Mankiw (2002) find

$$\pi_t - \pi_{t-1} = 3.8 - 0.63 u_t + \varepsilon_t$$

(7)

implying a NAIRU of 6.1 percent\(^3\).

Many economists (including the ones providing the above regression results) have questioned whether it is appropriate to consider the NAIRU as being constant. Monetary regime changes and other changes in the economy would likely affect the NAIRU. Empirically one possibility for handling this is to allow $c$ to vary, i.e. estimate

$$\pi_t - \pi_{t-1} = c_t - \alpha u_t + \varepsilon_t,$$  

(8)

where $c_t$ is estimated according to some smoothing procedure. Ball and Mankiw (2002) do just that. They set $\alpha$ in equation (8) to 0.63 based on the estimate in (7) and estimate $c_t$ according to the Hodrick-Prescott filter (Hodrick and Prescott, 1997). Their estimates are sensitive to choice of smoothing parameter, but generally indicate the NAIRU peaking around 1980 and declining subsequently. One of Ball and Mankiw’s estimates indicates the NAIRU having a high-point at 6.8 percent in 1979 and then falling to 4.9 percent in 2000.

\(^3\) Actually $3.8/0.63 = 6.0$. The Blanchard and Katz (1997) calculation is 6.1, which may have been subject to less rounding error.
Staiger, Stock, and Watson (1997) and Gordon (1997) also provide time-varying estimates for the NAIRU, but in their regressions in explaining inflation they utilize variables for supply shocks and various lags of unemployment and inflation. The pattern of the NAIRU peaking around 1980 and then declining has been also noted in these studies. According to Gordon (1997) the NAIRU has decreased in the later years because of global competition, immigration, other factors weakening labour’s bargaining position, and the continuous decrease in prices of computers and other electronics. Ball and Mankiw (2002) suggest that an increase in the growth rate of labor productivity explains a large part of the decline in the US NAIRU during the late 1990s.

3. **Empirical Methodology in Current Paper**

With monetary regime changes and other changes in the economy, there is no reason to expect the NAIRU to be constant, so providing time varying estimates for it seems appropriate. By the same reasoning, however, there is no reason to expect the unemployment rate-inflation rate trade-off (the slope parameter, \( \alpha \)) to be constant over time, so time-varying estimates for \( \alpha \) would appear likewise appropriate. With that in mind, in this paper we estimate for US annual data 1951 – 2001 the following system of equations by utilizing a Kalman Filter:

\[
\begin{align*}
\pi_t - \pi_{t-1} &= c_t - \alpha_t u_t + \epsilon_t \\
c_t &= c_{t-1} + e_{1t} \\
\alpha_t &= \alpha_{t-1} + e_{2t}
\end{align*}
\]

where \( \epsilon_t \), \( e_{1t} \), and \( e_{2t} \) are assumed to be white noise processes in which \( e_{1t} \) and \( e_{2t} \) are allowed to be contemporaneously correlated, but \( \epsilon_t \) is independent of \( e_{1t} \) and \( e_{2t} \).

Following the pattern in equation (5), the time-varying NAIRU is estimated as

\[
NAIRU_t = \frac{\hat{c}_t}{\hat{\alpha}_t}
\]

where \( \hat{c}_t \) is the estimated time-varying intercept in equation system (9), and \( \hat{\alpha}_t \) is the estimated time-varying slope parameter with respect to unemployment in that equation system.

---

4 For a formal derivation of the Kalman filter and its advantages the interested reader is referred to Hatemi-J (2002) and references therein.
4. Estimation Results

We first tested each variable for unit roots by applying the standard augmented Dickey-Fuller unit root test and also Phillips-Perron tests for unit roots. The results, not reported but available on request, show that the null hypothesis of one unit root for inflation and the unemployment rate could be rejected at the conventional significance levels. We proceeded to estimate the parameters in equation system (9). Our Kalman-filter estimates for the time-varying parameters are presented in Figure 1.

**Figure 1 - Estimates for $c_t$**

As shown in Figures 1 and 2, both $c_t$ and $\alpha_t$ rise in the early period until the late 1970s – early 1980s, during which they peak, and thereafter fall. The $c_t$ pattern is more pronounced than that of the $\alpha_t$ pattern. The implication about the $\alpha_t$ pattern is that in the period prior to the late 1970s, inflation was becoming more sensitive to unemployment; to reduce unemployment by a certain percentage, policymakers would need to sacrifice more inflation than previously. How much inflation policymakers must sacrifice to reduce unemployment by a particular percentage peaked in 1982 according to the estimates and declined slowly thereafter.

These estimates for $c_t$ and $\alpha_t$ are used to estimate the time-varying NAIRU by applying equation (10). This time-varying NAIRU is presented in the following Figure 3 along with the actual unemployment rate. The NAIRU is estimated to generally rise and fall over this period, peaking in 1979. This is consistent with other studies of the time-varying NAIRU, which often find the NAIRU peaking around 1980. In the late 1990s and in 2000, unemployment falls below the NAIRU, implying pressure for inflation to increase, but by 2001 it was very close to the NAIRU, so inflation had little reason to change based on aggregate unemployment.
As shown in Figures 1 and 2, both $c_t$ and $\alpha_t$ rise in the early period until the late 1970s – early 1980s, during which they peak, and thereafter fall. The $c_t$ pattern is more pronounced than that of the $\alpha_t$ pattern. The implication about the $\alpha_t$ pattern is that in the period prior to the late 1970s, inflation was becoming more sensitive to unemployment; to reduce unemployment by a certain percentage, policymakers would need to sacrifice more inflation than previously. How much inflation policymakers must sacrifice to reduce unemployment by a particular percentage peaked in 1982 according to the estimates and declined slowly thereafter.

These estimates for $c_t$ and $\alpha_t$ are used to estimate the time-varying NAIRU by applying equation (10). This time-varying NAIRU is presented in the following Figure 3 along with the actual unemployment rate. The NAIRU is estimated to generally rise and fall over this period, peaking in 1979. This is consistent with other studies of the time-varying NAIRU, which often find the NAIRU peaking around 1980. In the late 1990s and in 2000, unemployment falls below the NAIRU, implying pressure for inflation to increase, but by 2001 it was very close to the NAIRU, so inflation had little reason to change based on aggregate unemployment.

**Figure 3 - The Time Varying NAIRU and the Unemployment Rate**

Figure 4 presents the time series for the actual unemployment rate minus the NAIRU estimate and the time series for the change of inflation. We see that there is a very strong negative relationship between the two series, indicating the Phillips curve relationship. The last three years match our expectations noted in the previous
paragraph. In 1999 and 2000 the unemployment rate was notably below the NAIRU estimate, and likewise inflation was notably increasing. By 2001, however, unemployment was very close to the NAIRU estimate, and, as expected, inflation did not change much.

**Figure 4** - Unemployment Rate Minus NAIRU and the Change in Inflation

![Figure 4](image)

5. **Conclusions**

This study has dealt with the potential relationship between inflation and the unemployment rate in the US economy. We find annual estimates for the NAIRU over the 1951-2001 period. We base these estimates on a time-varying parameter model, which is estimated by the Kalman filter. Ours is not the first attempt to estimate a time-varying NAIRU, and our results do not differ significantly from those of others who have provided such estimates: the NAIRU is generally rising until around 1980 (peaking in 1979 in our case) and is generally falling thereafter.

However, there is an interesting associated issue when estimating the NAIRU: how sensitive is inflation to unemployment? Previous studies have estimated this as constant when estimating the NAIRU as time varying. We consider that also allowing the sensitivity of inflation to the unemployment rate to be time varying makes sense, as it follows from the same logic that leads us to think the NAIRU is likely to be time-varying. Monetary regime changes and other changes in the economy over time are likely to cause the NAIRU to change over time, and likewise they may also affect the sensitivity of inflation to the unemployment rate over time.
In our estimates, we therefore allow the sensitivity of inflation to the unemployment rate to be time varying along with the NAIRU. Inflation is found to have become increasingly sensitive to unemployment in the late 1950s through the early 1970s, and peaked in the late 1970s – early 1980s. After that, the sensitivity decreased only slightly.

R. SCOTT HACKER
Jönköping International Business School, Jönköping, Sweden

ABDULNASSER HATEMI-J
University of Skövde, Department of Economics, Skövde, Sweden and Dohuk University, Department of Economics, Iraqi Kurdistan

REFERENCES
The objective of this study is to provide estimates of the Phillips curve in the US during the period 1951-2001 using some time-varying parameters and the Kalman filter. Time-varying estimates for the sensitivity of inflation to the unemployment rate are provided in addition to time-varying estimates for the NAIRU (the non-accelerating inflation rate of unemployment). Our results for the NAIRU do not significantly differ from that of others with time-varying estimates of it, with it peaking around 1980 (1979 in our case). Inflation is found to have become increasingly sensitive to unemployment in the late 1950s through the early 1970s, and peaked in the late 1970s – early 1980s. After that, the sensitivity decreased only slightly.

Keywords: Phillips curve, Time varying NAIRU, Kalman filter

JEL Classification: E10, C32

ABSTRACT


RIASSUNTO

Stime ‘Time-Varying’ per il tasso naturale di disoccupazione e la curva di Phillips negli U.S.A. attraverso l’uso del filtro di Kalman