DIRECTION ACCURACY, FORECASTING ERROR AND THE PROFITABILITY OF CURRENCY TRADING: SIMULATION-BASED EVIDENCE

1. Introduction

Economists have been preoccupied by the Meese-Rogoff (1983) puzzle, that exchange rate models cannot outperform the random walk in exchange rate forecasting. For example, Leitch and Tanner (1991) point out that economists are bewildered by the observation that profit-maximizing firms buy professional forecasts when the naïve random model forecasts better or just as well as any model. Typically forecasting accuracy is measured by the root mean square error (RMSE) and similar metrics that only take into account the (absolute) value of the forecasting error (that is, the difference between the forecast and actual values of the exchange rate). However, some economists argue that a correct prediction of the direction of change is more important than the magnitude of the error, while others have suggested that the ultimate test of forecasting power is the ability to make profit by trading on the basis of the forecasts.

Cheung *et al.* (2005) argue that minimizing the RMSE may not be important from an economic standpoint and that the direction of change is

> "perhaps more important from a market timing perspective".

Leitch and Tanner (1991) argue that the direction of change may be more relevant for profitability and economic concerns. This proposition is justified on the grounds that statistical measures of forecasting accuracy (such as the RMSE) bear a very weak relation to the profit generated by acting on the basis of the forecasts and that the only substitute criterion for profit is a measure of direction accuracy. They further suggest that if profits are not observable, direction accuracy of the forecasts may be used as the evaluation criterion. Abhyankar *et al.* (2005) associate the ability to predict the direction of change with "economic value", which is a general term for profitability. While they argue that

"there are many different ways to characterize or define economic value",

they follow West *et al*. (1993) by using a Bayesian framework to study asset allocation. Their framework for measuring profitability involves a buy-and-hold strategy and the allocation of wealth between two assets that are identical in all aspects except for the currency of denomination. As a measure of profitability they use the wealth ratio, which is the ratio of end-of-period wealth accumulated by using fundamentals and random walk to the initial wealth.

However, whether the prediction of the magnitude of change is more or less important than the prediction of the direction of change depends on the underlying situation. For example, in intra-day currency trading, where the interest rate differential is ignored, the prediction of the direction of change is the only thing that matters. When interest rates are taken into account, both factors become important because the decision depends on the expected rate of return, which consists of the interest rate differential and expected change in the exchange rate. When the underlying situation involves speculation on combined currency option positions, such as a straddle, the only thing that matters is the magnitude of change (irrespective of the direction).

The objective of this paper is to find out if profitability is more strongly associated with the ability to predict the direction of change than the magnitude of the forecasting error. For this purpose we use a simulation exercise to generalize the results rather than using historical data on specific exchange rates, which may produce specific results. In designing the simulation exercise actual data provide guidelines for the determination of the relative scales and volatilities of interest differentials and changes in exchange rates.

2. Methodology and theoretical considerations

In forecasting-based trading a decision is taken to go long on one currency and short on another on the basis of the forecast or expected return. Let i_a be the interest rate on currency a, i_b the interest rate on currency b and S is the exchange rate between the two currencies measured as the price of one unit of b in terms of a. If $\hat{\hat{S}}_{t+1}$ is the forecast percentage change in the exchange rate then by taking a long position on b and a short position on a the forecast return is

$$\hat{\pi} = (i_b - i_a) + \hat{\hat{S}}_{t+1} \tag{1}$$

If $\hat{\pi} > 0$, this position will be taken in which case the realised return will be

$$\pi = (i_b - i_a) + \dot{S}_{t+1} \tag{2}$$

Otherwise the positions will be reversed in which case the realised return will be

$$\pi = (i_a - i_b) - \dot{S}_{t+1} \tag{3}$$

This operation is conducted on a period-by-period basis where a period extends between t and t+1. Profitability is measured by the mean return, which is calculated as

$$\overline{\pi} = \frac{1}{n}\sum_{t=1}^{n} \pi_t \tag{4}$$

We use the conventional measure of direction accuracy

$$DA = \frac{1}{n}\sum_{t=1}^{n} a_t \tag{5}$$

where

$$a_t = \begin{cases} 1 \\ 0 \end{cases} \quad \text{if} \quad \begin{cases} (\hat{S}_{t+1} - S_t)(S_{t+1} - S_t) > 0 \\ (\hat{S}_{t+1} - S_t)(S_{t+1} - S_t) < 0 \end{cases} \tag{6}$$

The root mean square error is calculated as

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(\frac{\hat{S}_t - S_t}{S_t}\right)^2} \tag{7}$$

which may also be written as

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\hat{\dot{S}}_t - \dot{S}_t)^2} \tag{8}$$

As we can see from equation (8), it does not matter for the calculation of the RMSE whether the forecasting error ($\hat{\dot{S}} - \dot{S}_t$) is positive or negative, which means that the RMSE does not take into account the direction of change. In essence, therefore, a perfect forecast is indicated by the RMSE when the following condition is satisfied:

$$\left|\hat{\dot{S}}_{t+1}\right| = \left|S_{t+1}\right| \tag{9}$$

It can be demonstrated that this condition is not necessarily associated with a profitable trade. Since interest rates are known at time t, the outcome (as far as profitability is concerned) depends on the percentage change in the exchange rate, which means that the deviation of the expected return from the actual return depends on the deviation of the forecast exchange rate from the actual value. Thus

$$\hat{\pi} - \pi = (i_b - i_a) + \hat{\dot{S}}_{t+1} - (i_b - i_a) - \dot{S}_{t+1} = \hat{\dot{S}}_{t+1} - \dot{S}_{t+1} \tag{10}$$

The condition represented by (10) can be satisfied either when $\hat{\dot{S}}_{t+1} = S_{t+1}$ or when $\hat{\dot{S}}_{t+1} = -S_{t+1}$. If $\hat{\dot{S}}_{t+1} = S_{t+1}$, then $\hat{\pi} - \pi = 0$, and since the decision is to take the underlying position only if $\hat{\pi} > 0$, this means that a zero prediction error is associated with profitability. If, on the other hand, $\hat{\dot{S}}_{t+1} = -S_{t+1}$, then

$$\hat{\pi} - \pi = 2\dot{S}_{t+1} \tag{11}$$

or

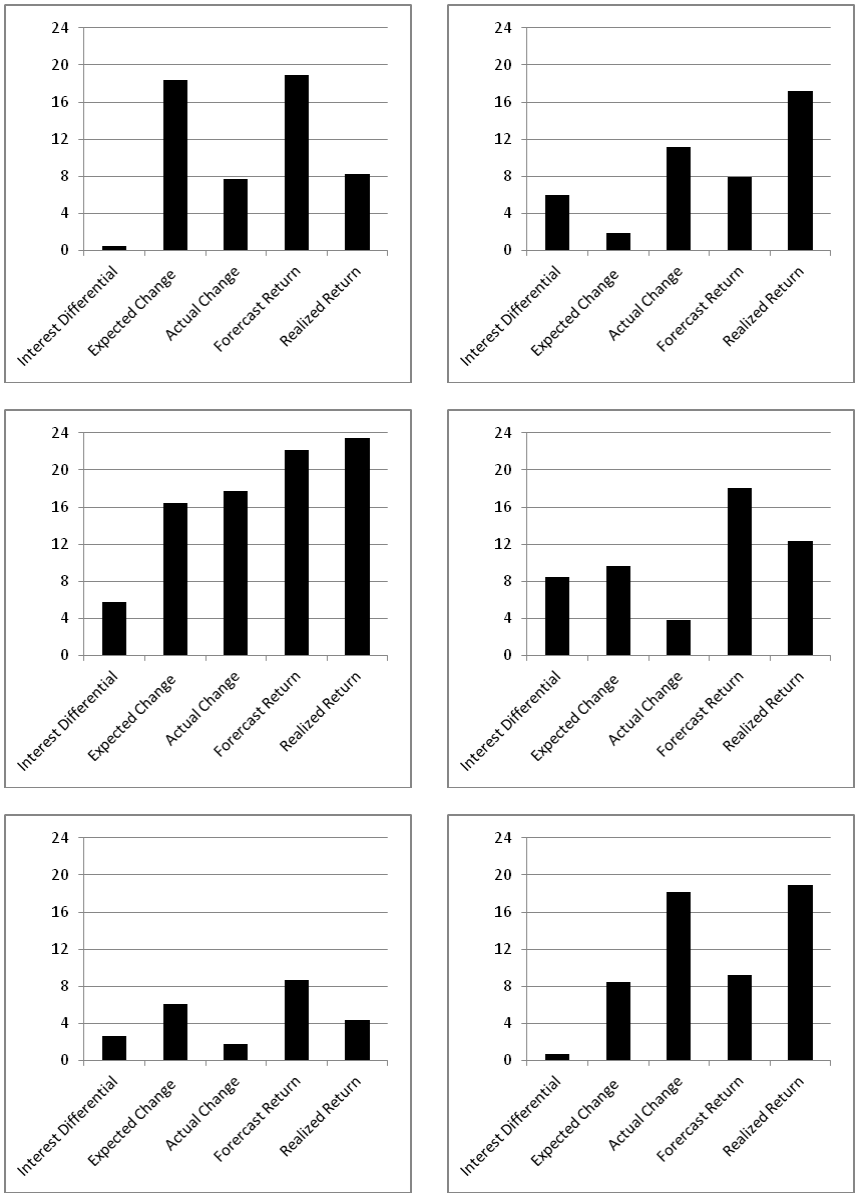
$$\pi = \hat{\pi} + 2\dot{S}_{t+1} \tag{12}$$

If $\dot{S}_{t+1} < 0$ and $2|\dot{S}_{t+1}| > \hat{\pi}$, it follows that $\pi < 0$. In this case a zero forecasting error (in the sense used to calculate the RMSE) is associated with a loss. Thus a model with a low (absolute) forecasting error may not lead to profitable trading based on the forecasts generated by the model. Correlation between profitability and the RMSE could be anything.

Let us now consider correlation between profitability and the ability to predict the direction of change. For a single trade, a correct prediction of the direction of change means that $\hat{\dot{S}}_{t+1}$ and $\dot{S}_{t+1}$ have the same signs, or that $\hat{\dot{S}}_{t+1}\dot{S}_{t+1} > 0$. It can be demonstrated that a satisfaction of this condition does not necessarily lead to a profitable trade, and vice versa. Some possibilities are exhibited in Table 1 and Figures 1 and 2. Take, for example, the case when $(i_b - i_a) > 0$, $\hat{\dot{S}}_{t+1} > 0$, and $\dot{S}_{t+1} > 0$, which produces $\hat{\pi} > 0$ and $\pi > 0$. In this case a correct prediction of the direction of change is always associated with a profitable trade, hence no additional conditions are needed under Case 1 in Table 1. In Figure 1, it is shown that the return is always positive irrespective of the absolute numerical values of the parameters determining the forecast and realised returns.

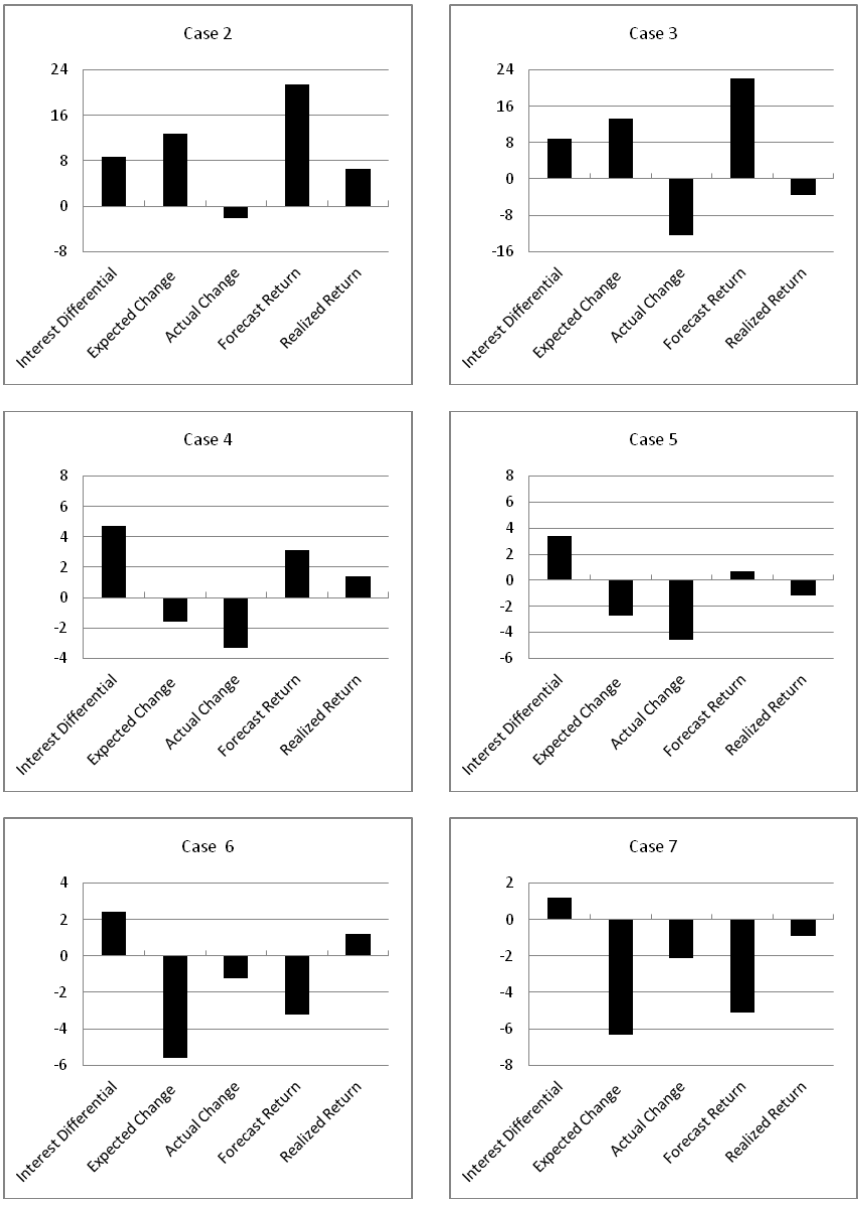
Consider now cases 2-7 in Table 1 and Figure 2 where a correct prediction of direction may be associated with a loss and vice versa. Consider Case 2, when the model predicts a positive change in the exchange rate and the actual change turns out to be negative. In this case we have $(i_b - i_a) > 0$, $\hat{\dot{S}}_{t+1} > 0$, and $\dot{S}_{t+1} < 0$, which produces $\hat{\pi} > 0$. Despite the wrong prediction of the direction of change, the trade will be profitable if $(i_b - i_a) > 0$ $\dot{S}_{t+1}$, otherwise the trade will produce a loss as in Case 3. On the other hand it can be shown that a correct prediction of direction may be associated with profit or loss. The possibilities exhibited in Table 1 and Figure 2 mean that profit or loss can be made with a correct or incorrect prediction of direction. Hence we should expect correlation between profitability and direction accuracy to be less than perfect. It remains to be seen

FIGURE 1 - *Correct Prediction of Direction is Always Profitable (Case 1)*



if the RMSE or direction accuracy is more strongly correlated with profitability. This is what we will find out in the following section based on simulated data.

FIGURE 2 - *Correct Prediction of Direction is not Always Profitable*

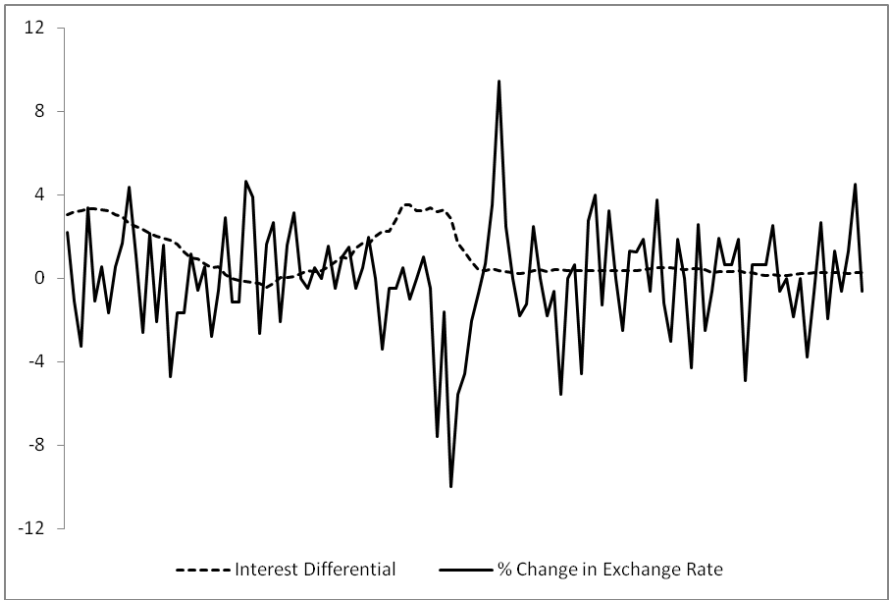


3. THE SIMULATION EXERCISE

The use of simulated rather than actual data is motivated by the desire to explore all possibilities, including changes in the sign

of the interest rate differential, which may not be found in actual data. Furthermore, simulated data allows us to explore a variety of scenarios that may not be available in historical data. However, actual data on the exchange rate between the dollar and pound (as a representative rate) over the period January 2004 and October 2013 are used to scale the parameters and assess their volatility for the purpose of generating simulated data. Figure 3 shows the interest rate differential between the dollar and pound as well as the percentage change in the exchange rate over the period between January 2004 and October 2013. Obviously the percentage change in the exchange rate is more volatile and of a greater order of magnitude than the interest differential. This difference is taken into account when the simulation exercise is conducted.

FIGURE 3 - *Interest Rate Differential and the Percentage Change in the USD/GBP Exchange Rate (January 2004-October 2013)*

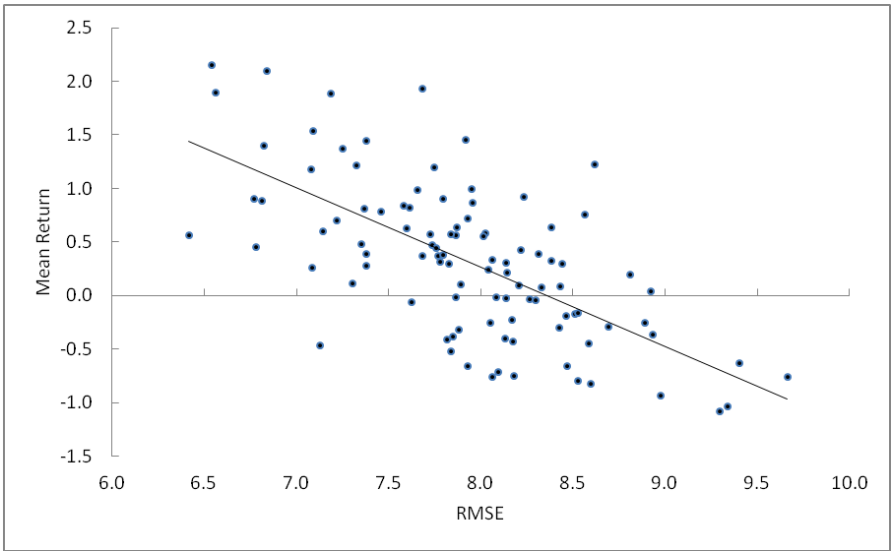


Based on the historical data, the interest rate differential is allowed to move within a range between -0.5% and 3.5% while the percentage change in the exchange rate is assigned a range between -10% and 9.5%. We start by generating observations on the interest rate differential as well as the forecast and actual changes in the

exchange rate. Accordingly the forecast return is calculated to determine the currency denominations of the short and long positions, and consequently the realised return. Also determined is whether or not the situation involves a correct prediction of direction, which is defined by the condition $\dot{S}_{t+1}\hat{S}_{t+1}>0$. For the purpose of this exercise 100 observations are generated on these variables, from which the RMSE and direction accuracy are calculated.

The objective of this exercise is to find out if direction accuracy is more strongly (and positively) correlated with profitability than the root mean square error. The answer to this question lies in Figures 4 and 5. In Figure 4, we find that the RMSE is negatively correlated

FIGURE 4 - *Correlation between Profitability and the Root Mean Square Error*



with profitability measured by the mean return. This result is not general in the sense that any simulation exercise or actual data produce negative correlation. The result indicates that profitability is so weakly dependent on the magnitude of the forecasting error that the association could turn negative. This is due to the fact that the RMSE does not take into account the sign of the forecasting error, regarding as a perfect forecast a situation when the actual and forecast values are equal with opposite signs. On the other hand,

Figure 5 shows strong positive association between profitability and direction accuracy. Correlation is not perfect because there are situations when a correct prediction of the direction of change is associated with loss, and vice versa (see Table 1).

FIGURE 5 - *Correlation between Profitability and Direction Accuracy*

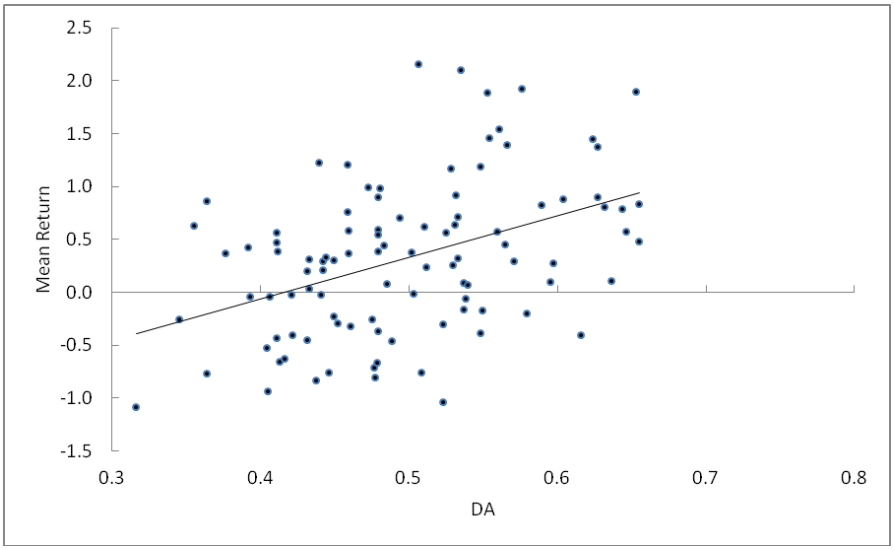


TABLE 1 - *The Conditions under which Direction Accuracy is Related to Trading Profit/Loss*

Case	$(i_b - i_a)$	$\hat{\dot{S}}_{t+1}$	$\dot{S}_{t+1}$	$\hat{\pi}$	π	Additional Conditions
1	+	+	+	+	+	--
2	+	+	-	+	+	$(i_b - i_a) > \dot{S}_{t+1}$
3	+	+	-	+	-	$(i_b - i_a) < \dot{S}_{t+1}$
4	+	-	-	+	+	$(i_b - i_a) > \lvert\hat{\dot{S}}_{t+1}\rvert$ $(i_b - i_a) > \lvert\dot{S}_{t+1}\rvert$
5	+	-	-	+	-	$(i_b - i_a) > \lvert\hat{\dot{S}}_{t+1}\rvert$ $(i_b - i_a) < \lvert\dot{S}_{t+1}\rvert$
6	+	-	-	-	+	$(i_b - i_a) < \lvert\hat{\dot{S}}_{t+1}\rvert$ $\lvert(i_a - i_b) < \lvert\dot{S}_{t+1}\rvert$
7	+	-	-	-	-	$(i_b - i_a) < \lvert\hat{\dot{S}}_{t+1}\rvert$ $\lvert(i_a - i_b)\rvert > \lvert\dot{S}_{t+1}\rvert$

What is important for the purpose of this exercise is that profitability is more positively correlated with direction accuracy than with the absolute forecasting error and hence the RMSE. This finding is consistent with the proposition put forward by Leitch and Tanner (1991) that the RMSE and similar metrics bear a very weak relation to the profit generated by trading on the basis of the forecasts and that direction accuracy is the only substitute criterion for profit. There is no puzzle, therefore, in observing profit-maximising firms paying for forecasts.

4. Conclusion

The literature on the forecasting power of exchange rate models deals with the issue whether it is more appropriate to measure forecasting accuracy in terms of the ability of the model to predict the direction of change rather than the magnitude of the forecasting error as measured by the root mean square error. Some economists argue that since profitability is the ultimate test of forecasting accuracy, direction accuracy should be more important than the RMSE because it is more strongly correlated with profitability. Although a correct prediction of the direction of change is neither a necessary nor a sufficient condition for a profitable trade (unless the interest rate differential is ignored), the results of the simulation exercise described in this paper show that direction accuracy is indeed more strongly correlated with profitability than the root mean square error.

Imad Moosa
School of Economics, Finance and Marketing, RMIT, Victoria, Australia

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ABSTRACT

A simulation exercise is conducted to find out if the profitability of forecasting-based currency trading is more related to the ability of the underlying model to predict the direction of change than the magnitude of the forecasting error. Theoretical considerations show that a correct prediction of the direction of change is neither a necessary nor a sufficient condition for a profitable trade. The results of the simulation exercise indicate that profitability is more strongly correlated with direction accuracy than with the root mean square error.

Keywords: Direction Accuracy, Forecasting, Currency Trading, Simulation

JEL Classification: F31, F37

RIASSUNTO

Accuratezza direzionale, errore previsionale e convenienza del currency trading: evidenze dalle simulazioni

In questo studio viene condotta una simulazione al fine di verificare se la convenienza del *currency trading* basato su modelli previsionali è maggiormente riferibile alla capacità del modello di prevedere la direzione del tasso di cambio o piuttosto alla portata dell'errore di previsione. Considerazioni teoriche mostrano che una previsione corretta della direzione non è condizione né necessaria né sufficiente affinché l'operazione sia conveniente. I risultati della simulazione evidenziano che tale convenienza è maggiormente correlata con l'accuratezza della direzione che con l'errore quadratico medio.