TARIFF AND PUBLIC GOOD IN A SMALL OPEN GROWTH ECONOMY WITH EXTERNALITIES AND CONGESTION*

1. Introduction

This paper is concerned with effects of changes in economic policies such as tariff, and tax rates on wealth, incomes, consumption and production on economic structure and capital accumulation, and trade balance in small open economies. It has been well recognized that trade policies have played an important role in modern economic development. Krueger (1997) emphasizes the importance of studying trade policy in the following way:

> "… changing trade policy is among the essential ingredients if there is hope for improved economic performance".

A main concern of this study is how tariffs on the imported goods affect growth of a small open economy. Tariffs are instruments for affecting allocation of resources and the distributions of factor incomes. There are many formal models on the impact of tariffs on economic growth for small open economies. For instance, in his seminal work on the topic, Mundell (1961) shows that under flexible exchange rates a tariff is contractionary. In the Mundell model, a tariff raises the terms of trade, increasing savings. The rise in savings reduces aggregate demand and necessitates a fall in aggregate supply in order for the goods market to clear. Chan (1978) extends Mundell's model by adding a money market. The model also predicts that a tariff is contractionary. The subsequent Eichengreen (1981) develops a portfolio balance with imperfectly flexible wage, showing that a tariff increases output and employment initially, while reducing them subsequently. Kimbrough (1982) develops a small-open economic

* The author is grateful to the constructive comments of the anonymous referee. The author is also grateful for the financial support from the Grants-in-Aid for Scientific Research (C), Project No. 25380246, Japan Society for the Promotion of Science.

model with traded good and non-traded good. The study shows that the effect of the tariff on the current account balance is dependent on the complementarity or substitutability of the imported good and the non-traded good in consumption demand. Lin (1996) examines tariffs on different industries. Lin's model shows that tariff protection may harm the import-complementing sector more than it assists the import-substituting sector, thereby aggravating consumption distortions and reducing the welfare. Nevertheless, the early literature neglects endogenous capital accumulation. In the recent literature on the topic most of the studies on the impact of tariffs on economic growth are based on Ramsey-type models of capital accumulation. For instance, Sen and Turnovsky (1989) developed a one-sector model of a small open economy with consumption tariff. The model shows that a consumption tariff results in a reduction in employment and decumulation of capital. Lee (1993) builds a neoclassical growth model in which the government distributes its revenue as a lump-sum transfer to household. In Lee's model no trade intervention can improve economic growth. Osang and Pereira (1996) develop a small open growth model with physical and human capital with investment subsidy in addition to the lump-sum transfer as spending policy. Naito (2000) develops a model with domestic and foreign intermediate good, with the Marshallian externalities in the domestic intermediate goods sector. The model shows that if the government uses the tariff revenue for correcting the domestic distortion, then the impact is growth-improving. There are many other studies on the impact of tariffs on economic growth (e.g., Leung, 1999; Chen *et al.*, 2008; Ogawa, 2009; Lee, 2011: Nunn and Trefler, 2010; Felbermayr *et al.*, 2013; and Hwang and Turnovsky, 2013). Although our study is much influenced by the literature of small open economic growth with tariffs, we deviate from the literature in that, instead of the Ramsey-type utility function, we use an alternative approach by Zhang (1993) to modeling behavior of the household and we extend the economic structure of the two sector model by Uzawa (1961) to an open economy of capital accumulation with public goods, imported good, externalities, congestion, and different taxes and tariff.

Another issue in this study is related to the impact of disturbances such as global economic changes, terms of trade and prices of input factors and goods (e.g., Sachs, 1982; Svensson and Razin, 1983; and Matsuyama, 1987). For instance, in a newly enriched economy consumers may change their preference for foreign brands. It is significant to examine how such changes in the preference may affect the economy's growth and structure. A widely discussed topic in the literature is how a change in a country's terms of trade affects economic growth (Mendoza, 1995; Kose, 2002; and Turnovsky and Chattopadhyay, 2003). This study studies effects of the preference for foreign goods and changes in the rate of interest and the price of imported good on trade balance and economic growth. We demonstrate that under certain conditions a stronger desire for foreign brands may hurt economic growth. To properly deal with interactions among capital accumulation, economic structural change, trade balance, public goods, import, changes in terms of trade, externalities, and congestion, we need a genuine dynamic framework. Nevertheless, there are only a few dynamic economic models which address these interactions in an integrated framework. This study deals with the economic dynamics of a small open economy by developing a growth model with public good, externalities, and imported good, basing on the Uzawa two-sector growth model. The model is a combination of the basic features of a few well-known models in the literature of economic growth. These models include Solow's growth model (Solow, 1956), Uzawa's two-sector growth model (Uzawa, 1961), the neoclassical growth models of small-open economies, and the growth models with externalities, public good, and congestion.

This study follows the motion of the economic system from any steady at any point time to its steady state. Most of the recent contributions to the analysis of tariffs in the Ramsey-type framework concentrate on steady-state effects, ignoring transitional effects. As transitional analysis often involves the need to follow the motion of the system over a certain period of time, it is often associated with analytical difficulties. As pointed out by King and Rebelo (1993), the situation that the study of dynamics in neoclassical growth models with rational microfoundation has been largely unexplored still applies to most of the recent studies. This study will simulate the motion of the economic system. It should be also mentioned that this paper is an extension of the two models by Zhang (2009, 2010). This paper extends Zhang's models in that this model introduces imported goods and tariff on the imported good, while the models by Zhang do not consider tariff and imported goods. The reminder of the paper is organized as follows. Section 2 defines the growth model with an alternative approach to consumer behavior. Section 3 shows how we solve the dynamics and simulates the model. Section 4 examines effects of changes in some parameters on the economic system over time. Section 5 concludes the study. The appendix proves the main results in Section 3.

2. The growth model with public good and tariff

The economy is small and open and produces two goods: an internationally traded good (called industrial good) and a non-traded good (called services). To study terms of trade and foreign trade, like Eicher *et al.* (2008), we include another good, called imported good. The imported good is consumed by the domestic consumers, but not produced by the economy. Households consume the two goods and services. Capital is perfectly mobile in international market. The price of the industrial good is unity. The economy is so small that it has no impact on the interest rate r^* and price of the imported good p_Z. The rate of interest and the price of the imported good are constant. The capital good depreciates at a given rate, δ_k. All markets are perfectly competitive and capital and labor are completely mobile between the two sectors. There is no emigration and immigration. Labor is homogeneous and is fixed. The government collects taxes and tariff. The government's revenue is spent only on financing the public sector. The public sector supplies public services. Both producers and consumers are affected by public services. We use subscript index, i and s, to stand for respectively the industrial and service sectors. Let τ_i, τ_s, and τ_k, stand for, respectively, the fixed tax rates on the industrial output, the service output, and the interest income. We introduce $\bar{\tau}_x \equiv 1-\tau_x$, where $x=i,s,k$.

Industrial Sector

We use $K_j(t)$ and $N_j(t)$ to stand for the capital stock and labor force employed by sector j, $j = i, s$, at time t. We use $F_j(t)$ to represent the output level of sector j. The production function of the industrial sector is given by

$$F_j(t)=\Omega_i(t)K_i^{\alpha i}(t)N_i^{\beta i}(t),\ \alpha_i,\ \beta_i,\ >0,\ \alpha_i+\beta_i=1, \tag{1}$$

where α_i, and β_i are parameters and where $\Omega_i(t)$ is a function of public service, externalities, and congestion. In this study $\Omega_i(t)$ is specified as follows

$$\Omega_i(t)=A_i\,F_p^{\theta_{pi}}(t)K_i^{\theta_{ei}}(t)\left(\frac{K_p(t)}{K_i(t)}\right)^{\theta_{ci}},\ \ A_i,\theta_{pi},\theta_{ei},\theta_{ci}\ge 0$$

Like in Zhang (2010), the term $F_p^{\theta pi}$ measures the impact of public service on the industrial sector's productivity, the term $K_p^{\theta ei}$ reflects the effect of externalities, and the term $(K_p(t)/K_i(t))^{\theta ci}$ is the effect of congestion of public goods. If $\theta_{ei}=\theta_{ci}=0$, there is neither congestion nor externality (see Eicher and Turnovsky 2000; Liu and Turnovsky, 2005). This is a limited case as most of public services are subject to some degree of congestion. In this study the congestion effect is measured by the term, $\left(K_p(t)/K_i(t)\right)^{\theta ci}$. This term is interpreted as that for a fixed level of public capital, a rise in the private capital tends to reduce the efficiency of public services (see, Gómez, 2008). The markets are competitive. The labor and capital earn their marginal products, and firms earn zero profits. We use $w(t)$ to represent the wage rate. The marginal conditions for the industrial sector are

$$r_\delta = \alpha_i \, \bar{\tau}_i \, \Omega_i(t) k_i^{-\beta_i}(t), \;\; w(t) = \beta_i \, \bar{\tau}_i \, \Omega_i(t) k_i^{\alpha_i}(t), \tag{2}$$

where $k_i(t) \equiv K_i(t)/N_i(t)$, and $r_\delta \equiv r^* + \delta_k$.

Service Sector

The service production uses three inputs, the public service, capital stock $K_s(t)$, and labor force $K_s(t)$. The production function of the service sector is

$$F_s(t) = \Omega_s(t) K_s^{\alpha_s}(t) N_s^{\beta_s}(t), \;\; \alpha_s, \, \beta_s > 0, \;\; \alpha_s + \beta_s = 1, \tag{3}$$

where α_s and β_s are parameters and $\Omega_s(t)$ is a function of public service, externalities, and congestion. We specify $\Omega_s(t)$ as follows

$$\Omega_s(t) = A_s \, F_p^{\theta_{ps}}(t) K_s^{\theta_{es}}(t) \left(\frac{K_p(t)}{K_s(t)} \right)^{\theta_{cs}}, \;\; A_s, \theta_{ps}, \theta_{es}, \theta_{cs} \geq 0$$

Let $p(t)$ stand for the price of service. The marginal conditions for the service sector are

$$r_\delta = \alpha_s \, \bar{\tau}_s \, \Omega_s(t) p(t) k_s^{\alpha_s - 1}(t), \;\; w(t) = \beta_s \, \bar{\tau}_s \, \Omega_s(t) p(t) k_s^{\alpha_s}(t), \tag{4}$$

where $k_s(t) \equiv K_s(t)/N_s(t)$.

Behavior of Domestic Households

This paper models consumers' behavior in the approach proposed by Zhang (e.g., 1993). The current income is

$$y(t) = \bar{\tau}_k \, r^* \, \bar{k}(t) + w(t) \tag{5}$$

where $r^* k(t)$ is the interest payment and $w(t)$ the wage payment. The variable $y(t)$ is called the current income as it consists of the household's current wage and current payment from ownership of wealth. The sum of income that the household consumes and saves is not necessarily equal to the current income. A person who has no wage income and negligible interest payment may live well as the person sells some of the wealth. We assume that selling and buying wealth can be conducted instantaneously without any transaction cost. The total value of the wealth that a consumer can sell to purchase goods and to save is equal to $p_i(t)k(t)$ where $p_i(t)$ is the price of the industrial good. In our model $p_i(t) = 1$. The sum of the current income and the total value of the household's (disposable) wealth is called disposable income. The disposable income is the amount of money that the household can use to consume and to save. The disposable income is

$$\hat{y}(t) = y(t) + \bar{k}(t). \tag{6}$$

The disposable income is used for saving and consumption. At time t the consumer has the total amount of income equaling $\hat{y}$ to distribute between consuming and saving.

At each point of time, a consumer distributes the total available budget among the consumption of service $c_s(t)$, industrial good $c_i(t)$, imported good $c_Z(t)$, and saving $s(t)$. We use τ_T to stand for the fixed tariff rate on the imported good. The budget constraint is

$$(1 + \tilde{\tau}_s)p(t)c_s(t) + (1 + \tilde{\tau}_c)c_i(t) + (1 + \tau_T)p_Z c_Z(t) + s(t) = \hat{y}(t), \tag{7}$$

where $\tilde{\tau}_s$ and $\tilde{\tau}_c$ are respectively the tax rates on the consumption of service and industrial good. Equation (7) implies that consumption and saving exhaust the household's disposable income. The household's utility level, $U(t)$, is specified as follows

$$U(t) = \theta c_s^{\gamma_0}(t)c_i^{\xi_0}(t)c_Z^{\varsigma_0}(t)s^{\lambda_0}(t), \ \gamma_0, \xi_0, \varsigma_0, \lambda_0 > 0.$$

Here, the parameters γ_0, ξ_0, ς_0 and λ_0 are respectively the household's elasticity of utility with regard to the service, industrial good, imported good, and saving. We call γ_0, ξ_0, ς_0 and λ_0 to consume the service, to consume the industrial good, to consume the imported good, and to hold wealth, respectively. Maximizing $U(t)$ subject to (7) yields

$$c_s(t) = \frac{\gamma\, \hat{y}(t)}{p(t)}, \ c_i(t) = \xi\, \hat{y}(t), \ c_Z(t) = \varsigma\, \hat{y}(t), \ s(t) = \lambda\, \hat{y}(t), \tag{8}$$

where

$$\gamma \equiv \frac{\rho \gamma_0}{1 + \tilde{\tau}_s}, \ \xi \equiv \frac{\rho \xi_0}{1 + \tilde{\tau}_c}, \ \varsigma \equiv \frac{\rho \varsigma_0}{p_Z(1 + \tau_T)}, \ \lambda \equiv \rho \lambda_0, \ \rho \equiv \frac{1}{\gamma_0 + \xi_0 + \varsigma_0 + \lambda_0}.$$

According to the definition of $s(t)$, the wealth accumulation for the household is

$$\dot{\bar{k}}(t) = s(t) - \bar{k}(t) \tag{9}$$

This equation states that the change in wealth equals the saving minus the dissaving.

The Public Sector

We follow Zhang (2010) in modeling the public sector. The government supports the public sector by the government revenue. The production factors of the public sector are paid at the same rates that the private sectors pay the services of these factors. The input factors are used effectively by the public sector in the sense that the government revenue is spent in such a way that public services are maximized. The output of the public sector is dependent on capital input $K_p(t)$ and the labor force $N_b(t)$ as follows

$$F_p(t) = A_p\, K_p^{\alpha_{0p}}(t) N_p^{\beta_{0p}}(t), \ \ \alpha_{0p}, \beta_{0p}, A_p > 0. \tag{10}$$

Let $Y_p(t)$ represent the government revenue. In this study we assume that the tariff revenue is used for supplying the public good. Tariff revenue may be distributed in different ways. If the revenue is inefficiently used, it may reduce national growth. It can also enhance national growth as, for instance, high tariffs may protect national industries. For instance, in Krugman (1987) and Grossman and Helpman (1991), tariffs are applied to the industry subject to externalities. In these approaches tariffs can raise *per capita* gross domestic product. In our model, we assume that the tariff revenue is used to supply the public good, which directly increases productivities of the two sectors.

The government's revenue consists of the tariff revenue, and the taxes on the two sectors, the ownership of wealth, and consumption, i.e.

$$Y_p(t) = \tau_i\, F_i(t) + \tau_s\, p(t) F_s(t) + \tau_k\, r^* \bar{k}(t) N + \tilde{\tau}_s\, p(t) c_s(t) N + \tilde{\tau}_i\, c_i(t) N + \\ + \tau_T\, p_Z\, c_Z(t) N \tag{11}$$

As the government revenue is spent only on supplying public services, we have

$$w(t) N_p(t) + r_\delta\, K_p(t) = Y_p(t). \tag{12}$$

The budget constraint means that the government revenue spent on the workers and capital stocks employed by the public sector. Maximizing public services under the budget constraint yields

$$r_\delta\, K_p(t) = \alpha_p\, Y_p(t), \ \ w(t) N_p(t) = \beta_p\, Y_p(t), \tag{13}$$

in which

$$\alpha_p \equiv \frac{\alpha_{0p}}{\alpha_{0p} + \beta_{0p}}, \ \ \beta_p \equiv \frac{\beta_{0p}}{\alpha_{0p} + \beta_{0p}}.$$

Demand of and Supply for Services

The equilibrium condition for services is

$$c_s(t)N = F_s(t). \tag{14}$$

Full Employment of Capital and Labor

The total capital stocks employed by the country, $K(t)$, is employed by the three sectors. The full employment of labor and capital is represented by

$$K_i(t) + K_s(t) + K_p(t) = K(t), \quad N_i(t) + N_s(t) + N_p(t) = N \tag{15}$$

It should be noted that this study assumes that the system adjusts to changes in prices instantly. The study on the impact of tariffs on a small open economy by Gavin (1991) considers that it takes time for the production sector to adjust to changes in relative prices. Gavin's model highlights the differential effects of a tariff on income and relative prices in the short versus the long run. The slow adjustment implies that inputs may not be fully employed. Following the neoclassical growth theory, this study assumes fast adjustments in the monetary variables.

Trade Balance

Let $\bar{K}(t)$ stand for the total wealth owned by the country's population, that is, $\bar{K}(t) = \bar{k}(t)N$. The capital owned by the population is not necessarily equal to the level of capital stocks employed by the country. We use $E(t)$ to denote the balance of trade. We have

$$E(t) = r^*\left(\bar{K}(t) - K(t)\right). \tag{16}$$

We have thus built the dynamic growth model with public good and tariff on imported goods.

3. The dynamics of the national economy

Although the economic model contains many variables, we can show that the motion of the economic system is determined by a single differential equation with $z(t) \equiv \bar{k}(t)/k_i(t)$ as the variable. The following lemma describes the procedure of determining the motion of all the variables.

Lemma 1

The motion of $z(t)$ is given by

$$\dot{z}(t) = \overline{\Lambda}(z(t)), \tag{17}$$

in which $\overline{\Lambda}$ is a function of $z(t)$ defined in the appendix. We determine all the other variables as functions of $z(t)$ as follows: $k_i(t)$ by (A15) $\rightarrow$ $k_s(t)$ by (A2) $\rightarrow$ $k_p(t)$ by (A4) $\rightarrow$ $\bar{k}(t) = z(t)\Lambda(t)$ by (A12) $\rightarrow$ $K_p(t)$ by (A11) $\rightarrow$ $K_i(t)$ by (A10) $\rightarrow$ $K_s(t)$ by (A6) $\rightarrow$ $N_j(t) = K_j(t)/k_j(t)$, $j = i,s,p$ by (A15) $\rightarrow$ $\Omega_i(t)$ and $\Omega_s(t)$ by the specified forms $\rightarrow$ $F_j(t)$ by the specified forms $\rightarrow$ $p(t)$ by (A3) $\rightarrow$ $\hat{y}(t) = p(t)F_s(t)/\gamma N$ $\rightarrow$ $c_s(t)$, $c_i(t)$, $c_Z(t)$ and $s(t)$ by (8) $\rightarrow$ $K(t)$ by (14) $\rightarrow$ $E(t)$ by (15).

From Lemma 1, we can determine the motion of economic system at any point of time as unique functions of the new variable $z(t)$ and the other exogenous variables (the rate of interest, technology, and preference). After we determine the motion of $z(t)$ by (17), we determine the motion of the whole system. It should be noted that in a model of a small open economy of Turnovsky (1996), the equilibrium growth rates of domestic capital and consumption are determined largely independent. The former is determined by production conditions, the latter is determined primarily by tastes. In our model, the variables are not only determined by the production conditions and the international rate of interest, but also by tastes.

In the literature of international economics effects of changes in terms of trade have caused a great attention. For instance, according to Eicher *et al.* (2008),

> "Previous authors have specified the borrowing cost to increase with the nation's level of debt. This specification, together with a constant rate of time preference and inelastic labor supply, implies that terms of trade shocks have no dynamic effects. The only response is that consumption fully adjusts instantaneously, with the current account remained unchanged".

With the regard to the change in the price of the imported good, our study shows similar effects. To explain this, we first note that from the appendix we show that the disposable income $\hat{y}(t)$ is not affected by p_Z. The price of the imported good affects the system only through $c_Z(t) = \varsigma \hat{y}(t)$. Except on the consumption level of the imported good, a change in the price of the imported good has no effect on the other variables in the dynamic system. Taking derivatives of $c_Z(t) = \varsigma \hat{y}(t)$ with respect p_Z

$$\frac{d\, c_Z(t)}{d\, p_Z} = -\frac{c_Z(t)}{p_Z}.$$

As the price is changed, the consumption level of imported good adjusts instantaneously with the other variables unaffected. A well-known result obtained by Harberger (1950) and Laursen and Metzler (1950) is that a deterioration in the terms of trade would reduce real income, thereby reducing saving and investment to cause a deterioration of the current account balance. In our approach the Harberger-Laursen-Metzler effect is not observed. As pointed out by Eicher *et al.* (2008), the effect is sensitive to several key features of the economy. These include, for instance, preferences (e.g., Mansoorian, 1993; Ikeda, 2001) and international capital market imperfections (Obstfeld, 1982). In our approach, for instance, if we specify the utility function in an alternative form, the system may react differently.

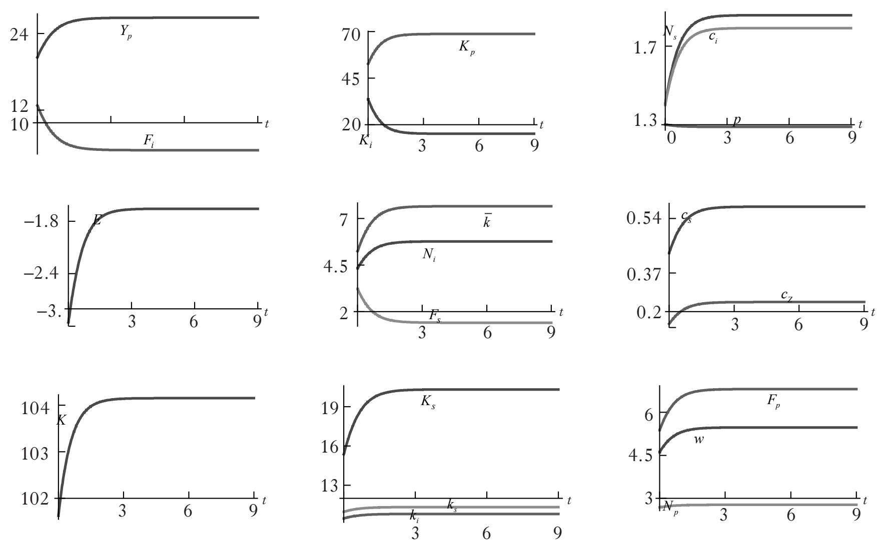
As the analytical expressions in our model are too tedious, it is difficult to get explicit conclusions. For interpretation, we simulate the model. We specify parameter values as follows

$$\begin{gathered} r^* = 0.06,\ \delta_k = 0.05,\ N = 10,\ A_i = 1.5,\ A_s = 1,\ \alpha_i = 0.3,\ \alpha_s = 0.31, \\ A_p = 0.9,\ \alpha_{0p} = 0.2,\ \beta_{0p} = 0.5,\ \lambda_0 = 0.6,\ \xi_0 = 0.15,\ \gamma_0 = 0.06, \\ \varsigma_0 = 0.04,\ \delta_k = 0.05,\ \theta_{pi} = 0.07,\ \theta_{ci} = 0.04,\ \theta_{ei} = 0.04,\ \theta_{ps} = 0.06, \\ \theta_{es} = 0.08,\ \theta_{cs} = 0.06,\ \theta_{is} = 0.03,\ p_Z = 1.3,\ \tilde{\tau}_s = 0.03, \\ \tilde{\tau}_c = 0.03,\ \tau_T = 0.08,\ \tau_k = 0.1,\ \tau_i = 0.03,\ \tau_s = 0.03. \end{gathered} \tag{18}$$

The rate of interest is fixed at 6 per cent and the population is 10. The propensity to save is 0.6. The propensity to consume is 0.15, which is higher than the propensity to consume services and propensity to consume the imported good. It should be remarked that although the specified values are not based on empirical observations, the choice does not seem to be unrealistic. For instance, some empirical studies on the US economy demonstrate that the value of the parameter, α, in the Cobb-Douglas production is approximately equal to 0.3. We specify α_j approximately equal to 0.3. The tariff rate is 8 per cent. The consumption tax rates on the good and service are 3 per cent. The property income tax rate is 10 per cent. The tax rates on the production sectors are 3 per cent. The price of the imported good is 1.3. The externalities, congestion effect and effects of the public good are not very strong. The initial condition is $z(0) = 0.5$. The motion of the dynamic system is plotted in Figure 1. The public expenditure and the output and two inputs of the public sector rise over time. The price of service falls slightly and the trade balance is improved. The outputs of the two sectors fall.

From Figure 1 we see that all the variables remain constant in the long term. This implies that the system may approach an

FIGURE 1 - *The Motion of the National Economy*



equilibrium point. The simulation identifies the equilibrium values of the variables as follows

w=2.78, p=1.29, E=–1.66, K=104.15, $\bar{K}$=76.52, Y_p=26.45, N_i=1.40, N_s=1.79, N_p=6.81, K_i=15.13, K_s=20.31, K_p=68.71, k_i=10.81, k_s=11.34, k_p=10.09, F_i=5.72, F_s=5.77, F_p=5.47, c_i=1.86, c_s=0.58, c_z=0.28.

Moreover, the eigenvalue at the equilibrium point is –1.84. The unique equilibrium point is stable. This guarantees that comparative dynamic analysis by simulation can illustrate the effects of parameter changes both on the transitional paths and the steady state.

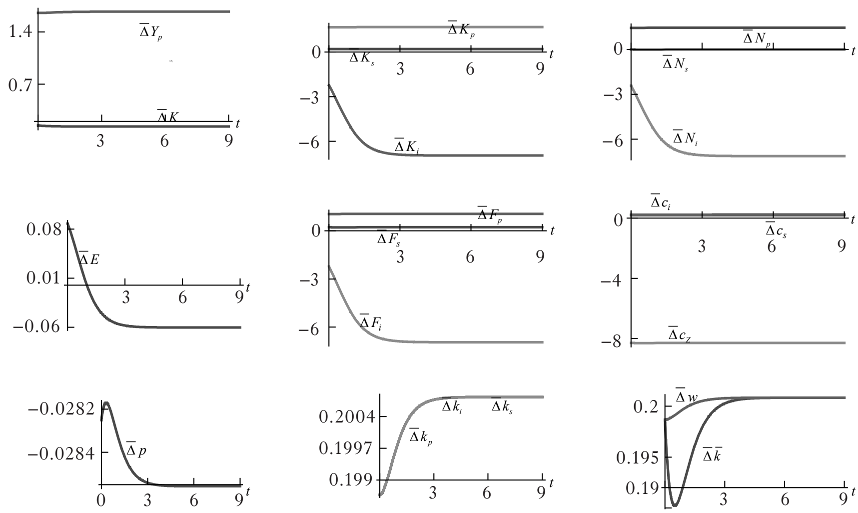
4. Comparative dynamic analysis

The previous section plots the motion of the variables. We now study how changes in parameters affect the motion of the national economy. As we have already known how to follow the dynamics of the entire system, it is straightforward to make comparative dynamic analysis. We introduce a variable, $\bar{\Delta}x(t)$, to stand for the change rate of the variable, $x(t)$, in percentage due to changes in the parameter value.

A Rise in the Tariff Rate

First, we examine the impact of the following change in the tariff rate on the imported good: τ_T:0.08⇒0.18. The rise in the tax rate reduces the consumption level of the imported and increases the revenue of the government. As the government spends more, the output and two inputs of the public sector are increased. Some of the labor force is shifted from the industrial sector to the public sector. The capital input and output level of the industrial sector are reduced. The price of the service is increased. The service sector's inputs and output are slightly affected. The consumption levels of the industrial good and the service are slightly increased. The trade balance is deteriorated initially but improved in the long term. The capital intensities of the three sectors, the wage rate and wealth per household are increased. In a Ramsey-type growth model of a small open economy with imported goods, Osang and Turnovsky (2000) demonstrate that a rise in the tariff on the imported good reduces the short-run growth rates of key economic variables and their common long-run equilibrium growth rate. In another model for a small open economy, Sen and Turnovsky show that a rise in the consumption tariff leads to a uniform decumulation in capital. Our model shows different results. In our approach with economic structure, externalities and public goods, as the government taxes more on the imported (consumption) good, the household saves more and spends more on the domestic good and service. As the increased revenue is used for providing public goods, the productivities are also increased. The increased savings and productivities result in higher wealth. In the literature of tariffs and economic growth there are opposite conclusions. In some modeling frameworks tariffs slow down economic growth (e.g., Jones and Manuelli, 1990; Easterly and Rebelo, 1993; Ben-David and Loewy, 1998; Naito, 2006a), while in some other models the conclusions are the opposite or situation-dependent (e.g., Krugman, 1987; Grossman and Helpman, 1990; Rivera-Batiz and Romer, 1991; Naito, 2005, 2006b). The empirical findings are also varied. There are positive relationships between tariff rates and economic growth (Harrison, 1996; and Edwards, 1992), not robust relationships (e.g., Sala-i-Martin, 1997), or positive relationships (Yanikkaya, 2003), or historically related relationships in a same economy (e.g., O'Rourke, 2000; and Clemens and Williamson, 2004).

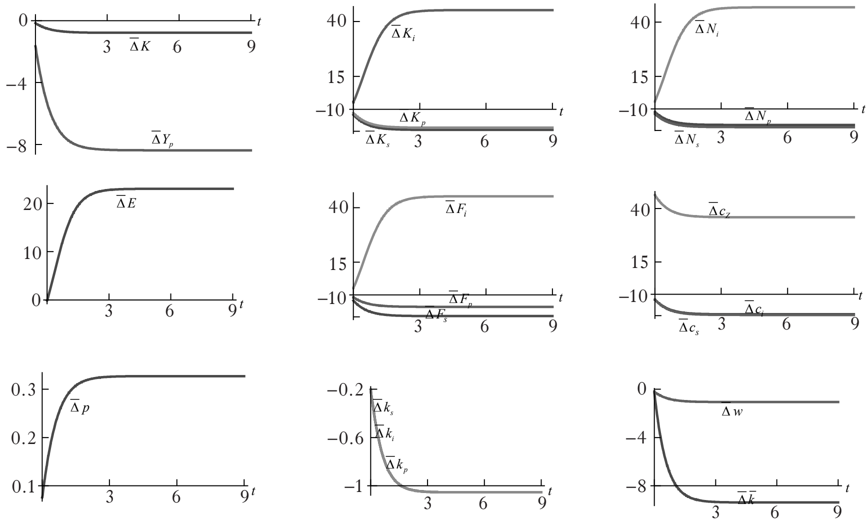
FIGURE 2 - *A Rise in the Tariff Rate*



A Rise in the Propensity to Consume the Imported Good

We now specify the following change in the propensity to consume the imported good: ς_0:0.04⇒0.06. The changes in the variables are plotted in Figure 3. As the propensity to consume the imported good is increased, the demand for the good is increased. The household spends more money on the consumption of foreign goods and less on the industrial good and service. The price of service is increased in association with falling in the production scale of service. As the household spends more on the imported good, the government revenue falls. The public sector produces less and employs less capital and labor inputs. The trade balance is deteriorated. The wage rate and capital intensities are reduced. The economy uses less capital and owns less wealth.

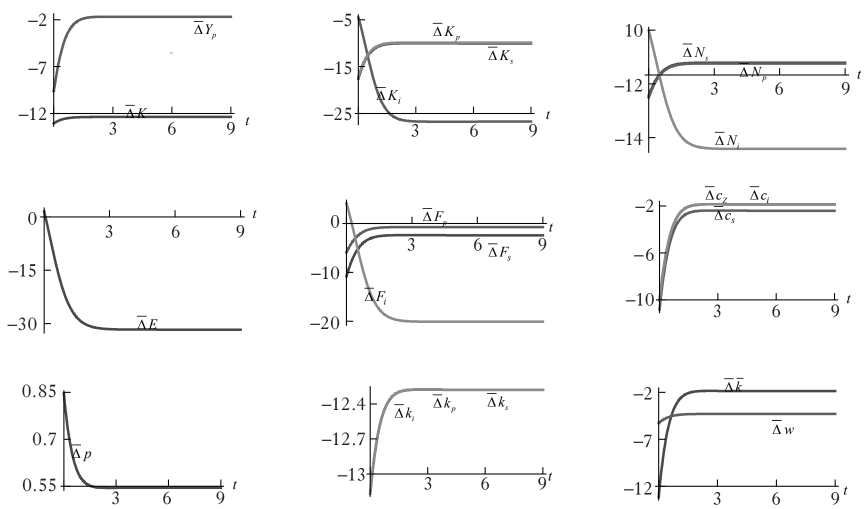
FIGURE 3 - *A Rise in the Propensity to Consume the Imported Good*



A Rise in the Rate of Interest in the International Market

We now allow the rate of interest in the global market to rise as follows: r^*:0.06⇒0.07. We plot the changes in the variables in Figure 4. As the costs of capital are increased, the capital intensities are reduced. The falling in the capital intensities leads to reduction in the wage rate. The three sectors use less capital in response to the rising cost of capital. The trade balance is improved. Labor force is shifted initially from the public and service sectors to the industrial sector but later from the industrial sector to the public and service sectors. The wealth per household and consumption levels of the good, service, and imported good fall are all reduced.

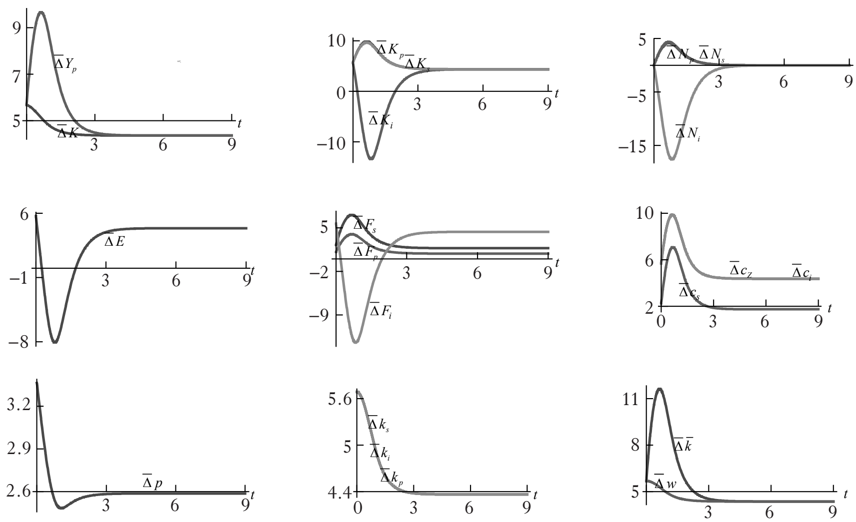
FIGURE 4 - *A Rise in the Rate of Interest*



The Externalities of the Industrial Sector being Strengthened

We now strengthen the externalities of the industrial sector as follows: θ_{ei}:0.06⇒0.07. The effects of the change in the parameter are illustrated in Figure 5. As the total productivity of the industrial sector is increased, the capital intensities of the three sectors are increased in the same rate. Here, we see that the change directions for some variables are different in the transitional processes and the steady state. As the productivity of the industrial sector is increased, initially some of the sector inputs are shifted to the other sectors in order to maintain the same rate of interest and wage rate. The government revenue is increased overtime. The national economy employs more capital and owns more wealth. The consumption levels of the good, service and imported good are all increased. The trade balance is deteriorated initially, improved late, and deteriorated in the long term. The price of the service is increased. The capital input and output level of the industrial sector are increased initially, reduced late, and increased in the long term.

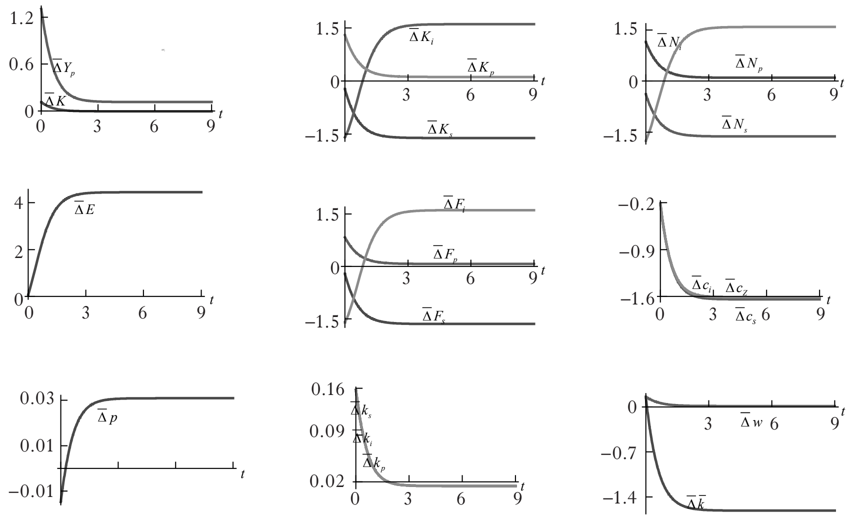
FIGURE 5 - *The Externalities of the Industrial Sector Being Strengthened*



A Rise in the Tax Rate on the Property Income

We now increase the tax rate on the property income as follows: τ_k:0.1⇒0.2. The effects of the change are plotted in Figure 6. The public expenditure is increased as the tax rate is increased. The public sector produces more and employs more capital and labor. The household tends to hold less wealth and consume less the good, service and imported good. The price of service and wage rate are slightly changed. The trade balance is deteriorated.

FIGURE 6 - *A Rise in the Tax Rate on the Property Income*



5. CONCLUSIONS

This paper developed an economic growth model of a small open economy with imported goods and tariff. The Uzawa-type two-sector model includes a public sector and takes account of externalities, congestion and effects of the public good on the productivities. The model is a combination of the basic features of a few well-known models in the literature of economic growth. These models include Solow's growth model, Uzawa's two-sector growth model, the neoclassical growth models of small-open economies, and the growth models with externalities, public good, and congestion. On the basis of the traditional literature of small open economies, the rate of interest is fixed in international market. The production side is the same as in the neoclassical growth theory. We used Zhang's utility function to determine saving and consumption. The study focuses on the effects of changes in the tariff rate, preference for foreign good, the global change in the rate of interest, the price of the imported good, the externalities of the industrial sector, on the dynamic paths of trade balance and economic growth. We simulated the model. The simulated economy has a unique stable equilibrium point. The comparative dynamic analysis provides some important insights. For instance, as the tariff rate is increased, the consumption level of the imported good is reduced; the government revenue, the output and two inputs of the public sector are all increased; some of the labor force is shifted from the industrial sector to the public sector; the capital input and output level of the industrial sector are reduced; the price of the service is increased and the service sector's inputs and output are slightly affected; the consumption levels of the industrial good and the service are slightly increased; The trade balance is deteriorated initially but improved in the long term; and the capital intensities of the three sectors, the wage rate and wealth per household are increased.

Wei-Bin Zhang
Ritsumeikan Asia Pacific University, Oita, Japan

REFERENCES

Ben-David, D. and M.B. Loewy (1998), "Free Trade, Growth, and Convergence", *Journal of Economic Growth,* 3(2), 143-170.

Chan, K.S. (1978), "The Employment Effects of Tariffs under a Free Exchange Rate Regime: A Monetary Approach", *Journal of International Economics*, 8(3), 414-424.

Chen, B.L., S.F. Lee and K. Shimomura (2008), *Import Tariffs and Growth in a Model with Habits*, in: Kamihigashi, L. Zhao (Eds), "International Trade and Economic Dynamics – Essays in Memory of Koji Shimomura", Springer-Verlag: Berlin.

Clemens, M.A. and J.G. Williamson (2004), "Why Did the Tariff-Growth Correlation Change After 1950?", *Journal of Economic Growth*, 9(1), 5-46.

Easterly, W. and S. Rebelo (1993), "Fiscal Policy and Economic Growth", *Journal of Monetary Economics*, 32(3), 417-458.

Edwards, S. (1992), "Trade Orientation, Distortions and Growth in Developing Countries", *Journal of Development Economics*, 39(1), 31-57.

Eichengreen, B.J. (1981), "A Dynamic Model of Tariffs, Output, and Employment under Flexible Exchange Rates", *Journal of International Economics*, 11(3), 341-359.

Eicher, T.S., S.F. Schubert and S.J. Turnovsky (2008), "Dynamic Effects of Terms of Trade Shocks: The Impact on Debt and Growth", *Journal of International Money and Finance*, 27(6), 876-896.

Eicher, T.S. and S.J. Turnovsky (2000), "Scale, Congestion, and Growth", *Economica*, 27(267), 325-346.

Felbermayr, G., B. Jung and M. Larch (2013), "Optimal Tariffs, Retaliation, and the Welfare Loss from Tariff Wars in the Melitz Model", *Journal of International Economics*, 89(1), 13-25.

Gavin, M. (1991), "Tariffs and the Current Account: On the Macroeconomics of Commercial Policy", *Journal of Economic Dynamics and Control*, 15(1), 27-52.

Gómez, M.A. (2008), "Consumption and Leisure Externalities, Economic Growth and Equilibrium Efficiency", *Scottish Journal of Political Economy*, 55(2), 227-249.

Grossman, G.M. and E. Helpman (1990), "Comparative Advantage and Long-Run Growth", *American Economic Review*, 80(4), 796-815.

Grossman, G.M. and E. Helpman (1991), Innovation and Growth in the Global Economy, MIT Press: Cambridge, MA.

Harberger, A.C. (1950), "Currency Depreciation, Income, and the Balance of Trade", *Journal of Political Economy*, 58(1), 47-60.

Harrison, A. (1996), "Openness and Growth: A Time Series, Cross-country Analysis for Developing Countries", *Journal of Development Economics*, 48(2), 419-447.

Hwang, Y-N. and S.J. Turnovsky (2013), "Exchange Rate Pass-through and the Effects of Tariffs on Economic Performance and Welfare", *Journal of International Money and Finance*, 33(1), 81-102.

Ikeda, S. (2001), "Weakly Non-separable Preferences and the Harberger-Laursen-Metzler Effect", *Canadian Journal of Economics*, 34(1), 290-307.

Jones, L.E. and R. Manuelli (1990), "A Convex Model of Equilibrium Growth", *Journal of Political Economy*, 98(5), 1008-1037.

Kimbrough, K.P. (1982), "Real Disturbances, the Current Account, and the Exchange Rate: The Case of a Tariff", *Journal of International Economics*, 13(3-4), 291-300.

King, R. and S. Rebelo (1993), "Transitional Dynamics and Economic Growth in the Neoclassical Model", *American Economic Review*", 83(4), 908-931.

Kose, M.A. (2002), "Explaining Business Cycles in Small Open Economies: 'How Much Do World Prices Matter?'", *Journal of International Economics*, 56(2), 299-327.

Krueger, A.O. (1997), "Trade Policy and Economic Development: How We Learn", *American Economic Review*, 87(1), 1-22.

Krugman, P. (1987), "The Narrow Moving Band, the Dutch Disease, and the Competitive Consequences of Mrs. Thatcher: Notes on Trade in the Presence of Dynamic Scale Economies", *Journal of Development Economics*, 27(1-2), 41-55.

Laursen, S. and L.A. Metzler (1950), "Flexible Exchange Rates and the Theory of Employment", *Review of Economics and Statistics*, 32(4), 281-299.

Lee, J.W. (1993), "International Trade, Distortions, and Long-Run Economic Growth", *Staff Papers – International Monetary Fund*, 40(2), 299-328.

Lee, S-F. (2011), "Tariff, Growth, and Welfare", *Review of International Economics*, 19(2), 260-276.

Leung, C.K.Y. (1999), "Income Tax, Property Tax, and Tariff in a Small Open Economy", *Review of International Economics*, 7(3), 541-554.

Lin, H.C. (1996), "Targeted Tariff Protection, Monopolistic Competition and Demand Interdependence", *International Economic Journal*, 10(2), 25-49.

Liu, W-F. and S.J. Turnovsky (2005), "Consumption Externalities, Production Externalities, and Long-run Macroeconomic Efficiency", *Journal of Public Economics*, 89(5-6), 1097-1129.

Mansoorian, A. (1993), "Habit Persistence and the Harberger-Laursen-Metzler Effect in an Infinite Horizon Model", *Journal of International Economics*, 34(1-2), 153-166.

Matsuyama, K. (1987), "Current Account Dynamics in a Finite Horizon Model", *Journal of International Economics*, 23(3-4), 299-313.

Mendoza, E.G. (1995), "The Terms of Trade, the Real Exchange Rate, and Economic Fluctuations", *International Economic Review*, 36(1), 101-137.

Mundell, R. (1961), "Flexible Exchange Rates and Employment Policy", *Canadian Journal of Economics and Political Science*, 27(4), 509-517.

Naito, T. (2000), "Tariff Revenue, Government Expenditure and Growth in a Small Open Economy", *The Japanese Economic Review*, 51(3), 391-406.

Naito, T. (2005), "Revenue-Neutral Environmental Tariff Reform, Growth, and Welfare", *Review of International Economics*, 13(5), 985-996.

Naito, T. (2006a), "Tariff and Tax Reform: Dynamic Implications", *Journal of International Economics*, 68(2), 504-517.

Naito, T. (2006b), "Growth, Revenue, and Welfare Effects of Tariff and Tax Reform: Win-Win-Win Strategies", *Journal of Public Economics*, 90(6-7), 1263-1280.

Nunn, N. and D. Trefler (2010), "The Structure of Tariffs and Long-Term Growth", *American Economic Journal: Macroeconomics*, 2(4), 158-194.

Obstfeld, M. (1982), "Aggregate Spending and the Terms of Trade: Is There a Laursen-Metzler Effect?", *Quarterly Journal of Economics*, 97(2), 251-270.

Ogawa, Y. (2009), "The Welfare Effects of Differential Tariff Rates", *Review of Development Economics*, 13(2), 348-355.

O'Rourke, K.H. (2000), "Tariffs and Growth in the Late 19th Century", *Economic Journal*, 110(463), 456-483.

Osang, T. and A.M. Pereira (1996), "Import Tariffs and Growth in a Small Open Economy", *Journal of Public Economics*, 60(1), 45-71.

Osang, T. and S.J. Turnovsky (2000), "Differential Tariffs, Growth, and Welfare in a Small Open Economy", *Journal of Development Economics*, 62(2), 315-342.

Sachs, J. (1982), "The Current Account in the Macroeconomic Adjustment Process", *Scandinavian Journal of Economics*, 84(2), 147-159.

Sala-i-Martin, X. (1997), "I Just Ran Two Million Regressions", *American Economic Review*, 87(2), 178-183.

Sen, P. and S.J. Turnovsky, (1989), "Tariffs, Capital Accumulation and the Current Account in a Small Open Economy", *International Economic Review*, 30(4), 811-831.

Solow, R. (1956), "A Contribution to the Theory of Economic Growth", *Quarterly Journal of Economics*, 70(1), 65-94.

Svensson, L.E.O. and A. Razin (1983), "The Terms of Trade and the Current Account: The Harberger-Laursen-Metzler Effect", *Journal of Political Economy*, 91(1), 97-125.

Turnovsky, S.J. (1996), "Fiscal Policy, Growth, and Macroeconomic

Performance in a Small Open Economy", *Journal of International Economics*, 40(1-2), 41-66.

Turnovsky, S.J. and P. Chattopadhyay (2003), "Volatility and Growth in Developing Economies: Some Numerical Results and Empirical Evidence", *Journal of International Economics*, 59(2), 267-295.

Uzawa, H. (1961), "On a Two-Sector Model of Economic Growth", *Review of Economic Studies*, 29(1), 47-70.

Zhang, W-B. (1993), "A Woman's Labor Participation and Economic Growth: Creativity, Knowledge Utilization and Family Preference", *Economics Letters*, 42(1), 105-110.

Zhang, W-B. (2009), "Agglomeration and Returns to Scale with Capital and Public Goods in a Multi-Regional Economy", *International Economic Journal*, 23(1), 81-109.

Zhang, W-B. (2010), "Regional Economic Geography with Externalities, Congestion, and Fiscal Policies in a Small-Open Growth Economy", *Journal of Geography Information System* 2(4), 201-209.

ABSTRACT

This paper develops a three-sector economic growth model of a small open economy with imported goods and tariff. The three are the industrial, service and public sectors. The model also introduces the impact of externalities, congestion and effects of the public good on the productivities. The model integrates a few well-known models in the literature of economic growth in a comprehensive framework. These models include Solow's growth model, Uzawa's two-sector growth model, the neoclassical growth models of small-open economies, and the growth models with externalities, public good, and congestion. The study focuses on the effects of changes in the tariff rate, preference for foreign good, the global change in the rate of interest, the price of the imported good, the externalities of the industrial sector, on the dynamic paths of trade balance and economic growth. The comparative dynamic analysis provides some important insights. For instance, the simulation demonstrates that as the tariff rate is increased, the consumption level of the imported good is reduced; the government revenue, the output and two inputs of the public sector are all increased; some of the labor force is shifted from the industrial sector to the public sector; the capital input and output level of the industrial sector are reduced; the price of the service is increased and the service sector's inputs and output are slightly affected; the consumption levels of the industrial good and the service are slightly increased; The trade balance is deteriorated initially but improved in the long term; and the capital intensities of the three sectors, the wage rate and wealth per household are increased.

Keywords: Taxes and Tariff, Trade Balance, Externalities, Congestion, Public Goods

JEL Classification: F11, O41

RIASSUNTO

Tariffe e beni pubblici in una piccola economia aperta in crescita con esternalità e congestione

In questo studio si esamina un modello a tre settori di piccola economia aperta con importazione di beni e con tariffe. I settori sono quello industriale, i servizi e il settore pubblico. Il modello considera anche l'impatto delle esternalità, della congestione e degli effetti dei beni pubblici sulla produttività, integrando così alcuni modelli di crescita già conosciuti in letteratura. Tra questi troviamo: il modello di crescita di Solow, il modello bisettoriale di Uzawa, i modelli neoclassici delle piccole economie aperte e i modelli di crescita con esternalità, beni pubblici e congestione. Questo modello è incentrato sugli effetti dei cambi nei prezzi, sulla preferenza verso i beni stranieri, sulle variazioni dei tassi di interesse a livello globale, sul prezzo dei beni importati, sulle esternalità del settore industriale, sulle dinamiche della bilancia commerciale e la crescita economica. L'analisi dinamica comparativa fornisce importanti risultati. Per esempio, la simulazione dimostra che, aumentando i prezzi, il livello dei consumi dei beni importati diminuisce; le entrate fiscali, l'output e due input del settore pubblico aumentano; parte della forza lavoro si sposta dal settore industriale a quello pubblico; il livello di input e di output del capitale del settore industriale si riduce; il prezzo dei servizi aumenta e l'input e l'output del settore dei servizi subiscono scarsa influenza; il livello dei consumi di beni industriali e dei servizi aumenta leggermente. La bilancia commerciale inizialmente peggiora per migliorare nel lungo periodo e gli investimenti nei tre settori, le retribuzioni e il benessere delle famiglie crescono.

APPENDIX
PROVING LEMMA 1

From (2), we get

$$\Omega_i = \frac{r_\delta \, k_i^{\beta_i}}{\alpha_i \, \bar{\tau}_i}, \quad w = r_w \, k_i, \tag{A1}$$

where $r_w \equiv \beta_i r_\delta / \alpha_i$. From (2) and (4), we have

$$k_s = \bar{\alpha} \; k_i \tag{A2}$$

where $\bar{\alpha} = \alpha_s \beta_i / \beta_s \, \alpha_i$. From (4), we solve

$$p = \frac{r_\delta}{\alpha_s \, \bar{\tau}_s \, \Omega_s \, k_s^{\alpha_s - 1}}. \tag{A3}$$

From (13) and (A1), we have

$$k_p = \tilde{r}_1 k_i, \tag{A4}$$

where $k_p \equiv K_p / N_p$ and $\tilde{r}_1 = \alpha_p r_w / r_\delta \, \beta_p$. Substitute $c_s = \gamma \hat{y} / p$ into (14)

$$\gamma \hat{y} N = p F_s .$$

Insert $r_\delta = \alpha_s \bar{\tau}_s \, p F_s / K_s$ in the above equation

$$\hat{y} N = n_0 K_s, \tag{A5}$$

where $n_0 \equiv r_\delta / \alpha_s \bar{\tau}_s \gamma$. Substitute the definition of $\hat{y}$ into (A5)

$$\tilde{r} \bar{k} N + \bar{\tau}_w r_w N k_i = n_0 K_s, \tag{A6}$$

where we use $w = r_w k_i$ and $\tilde{r} \equiv 1 + \bar{\tau}_k r^*$. From the definition of Y_p and $r_\delta K_p = \alpha_p Y_p$ in (13), we get

$$\frac{r_\delta}{\alpha_p} K_p = \frac{\tau_i \, r_\delta}{\alpha_i \, \bar{\tau}_i} K_i + \frac{\tau_s \, r_\delta}{\alpha_s \, \bar{\tau}_s} K_s + \tau_k \, r^* \, \bar{k} \, N + \left(\tilde{\tau}_s \, \gamma + \tilde{\tau}_i \, \xi + \tau_T \, p_Z \, \varsigma \right) N \, \hat{y} \tag{A7}$$

where we use $r_\delta = \alpha_i \bar{\tau}_i F_i / K_i$, $r_\delta = \alpha_s \bar{\tau}_s p F_s / K_s$ and (8). Insert (A5) in (A7)

$$K_p = h_i \, K_i + \frac{h_s \, \bar{\tau}_w \, r_w \, N}{n_0} k_i + h_k \, \bar{k} \tag{A8}$$

where we use (A6) and

$$h_i \equiv \frac{\alpha_p \, \tau_i}{\alpha_i \, \bar{\tau}_i}, \quad h_s \equiv \left[\frac{\tau_s}{\alpha_s \, \bar{\tau}_s} + \left(\tilde{\tau}_s \, \gamma + \tilde{\tau}_i \, \xi + \tau_T \, p_Z \, \varsigma \right) \frac{n_0}{r_\delta} \right] \alpha_p ,$$

$$p \quad h_k \equiv \frac{\alpha_p \, \tau_k \, r^* \, N}{r_\delta} + \frac{h_s \, \tilde{r} \, N}{n_0}.$$

Insert $k_j = K_j / N_j$ in (15)

$$\frac{K_i}{k_i} + \frac{K_s}{k_s} + \frac{K_p}{k_p} = N.$$

Substituting (A4) and (A2) into the above equation yields

$$K_i + \frac{K_s}{\overline{\alpha}} + \frac{K_p}{\tilde{r}_1} = N k_i \tag{A9}$$

Insert (A8) and (A6) in (A9)

$$K_i = h_1 k_i - h_2 \overline{k}, \tag{A10}$$

where

$$h_1 \equiv \left(1 + \frac{h_i}{\tilde{r}_1}\right)^{-1} \left(1 - \frac{h_s \overline{\tau}_w r_w}{\tilde{r}_1 n_0} - \frac{\overline{\tau}_w r_w}{\overline{\alpha} n_0}\right) N, \quad h_2 \equiv \left(1 + \frac{h_i}{\tilde{r}_1}\right)^{-1} \left(\frac{h_k}{\tilde{r}_1} + \frac{\tilde{r} N}{\overline{\alpha} n_0}\right)$$

Insert (A8) in (A10)

$$K_p = \tilde{h}_1 k_i + \tilde{h}_2 \overline{k}, \tag{A11}$$

where

$$\tilde{h}_1 \equiv h_i h_1 + \frac{h_s \overline{\tau}_w r_w N}{n_0}, \quad \tilde{h}_2 \equiv h_k - h_i h_2 .$$

From (A1) and the definition of Ω_i, we have

$$A_i F_p^{\theta_{pi}} K_i^{\theta_{ei} - \theta_{ci}} K_p^{\theta_{ci}} = \frac{r_\delta k_i^{\beta_i}}{\alpha_i \overline{\tau}_i}. \tag{A12}$$

Insert $N_p = K_p/k_p = K_p/\tilde{r}_1 k_i$ into $F_p = A_p K_p^{\alpha_{0p}} N_p^{\beta_{0p}}$ yields

$$F_p = \frac{A_p K_p^{\alpha_{0p} + \beta_{0p}}}{\tilde{r}_1^{\beta_{0p}} k_i^{\beta_{0p}}}. \tag{A13}$$

Substitute (A13) into (A12)

$$A_0 K_i^{\theta_{ei} - \theta_{ci}} K_p^{(\alpha_{0p} + \beta_{0p})\theta_{pi} + \theta_{ci}} = k_i^{\beta_i + \beta_{0p} \theta_{pi}}, \tag{A14}$$

where

$$A_0 \equiv \frac{\alpha_i \overline{\tau}_i A_i}{r_\delta} \left(\frac{A_p}{\tilde{r}_1^{\beta_{0p}}}\right)^{\theta_{pi}}.$$

Insert (A10) and (A11) in (A14)

$$k_i = \Lambda(z) \equiv \left[A_0 \left(h_1 - h_2 z\right)^{\theta_{ei} - \theta_{ci}} \left(\tilde{h}_1 + \tilde{h}_2 z\right)^{(\alpha_{0p} + \beta_{0p})\theta_{pi} + \theta_{ci}}\right]^{\beta_m} \tag{A15}$$

where $z \equiv \overline{k}/k_i$ and we require $\beta_m \equiv 1/(\beta_i - \alpha_{0p}\theta_{pi} - \theta_{ei}) \neq 0$. We determine k_i as a function of z. By the following procedure we can determine all the variables as functions of z: k_i by (A15) $\rightarrow$ k_s by (A2) $\rightarrow$ k_p by (A4) $\rightarrow$ $\overline{k} = z\Lambda$ by (A12) $\rightarrow$ K_p by (A11) $\rightarrow$ K_i by (A10) $\rightarrow$ K_S by (A6) $\rightarrow$ $N_j = K_j/k_j$, $j = i, s, p$ by (A15) $\rightarrow$ Ω_i and Ω_S by the specified forms $\rightarrow$ F_j by the specified forms $\rightarrow$ p by (A3) $\rightarrow$ $\overline{y} = pF_S/\gamma N$ $\rightarrow$ c_S, c_i, c_Z and s by (8) $\rightarrow$ K by (14) $\rightarrow$ E by (15).

It should be noted that from $\varsigma = \rho \varsigma_0 / p_Z (1+\tau_T)$, we see that (A7) is not affected by p_Z. It is straightforward to see that among all the variables only c_Z is affected by p_Z. By (9) and this procedure, we have

$$\dot{\bar{k}} = \Omega(z) \equiv \lambda \hat{y} - \bar{k}, \tag{A16}$$

where $\Omega(z)$, is a function of z. As the expression is tedious, we will not present it explicitly. Taking derivatives of (A15) with respect to time yields

$$\dot{\bar{k}} = \frac{d\Lambda}{dz} \dot{z}. \tag{A17}$$

We do not provide the expression of $d\Lambda / dz$ because it is tedious. Solve (A16) and (A17)

$$\dot{z} = \bar{\Lambda}(z) \equiv \Omega \left(\frac{d\Lambda}{dz} \right)^{-1} \tag{A18}$$

We thus proved Lemma 1.