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FACTOR SHARES, REDISTRIBUTION AND GROWTH IN A CAPTURED DEMOCRACY*

Abstract

In this paper, an endogenous growth model is presented, based on productive public expenditure and on a certain degree of inequality in the distribution of income, and of polarization in citizens' preferences concerning economic policy. The main innovation of this contribution consists in the political process that determines capital taxation, a process based on an "influence activity" exercised by the minoritarian capitalist class with the goal of capturing some political power from the majority of the citizens. In particular, lobbying activities investments, allow the capitalists to obtain a gradually lower level of capital taxation, to the benefit of themselves and of economic growth. Capital accumulation leads by the same token to an increment in lobbying activity, in order to convince the government to implement still a lower level of capital stock taxation, and more and more close to the preferences of the capitalists. In conclusion, it is demonstrated that in the long run, the full convergence obtains towards a political realm totally dominated by the few capitalists' de facto power, or to a political-economic reality similar to an "oligarchic technocracy" or to a "plutocracy."

Keywords: Political Economy; Government; Inequality; Economic Growth; Redistribution; Lobbying
JEL Classification: O11; O43

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Riassunto

Quote fattoriali, ridistribuzione e crescita in una democrazia catturata

In questo lavoro si presenta un modello di crescita endogena, basato sulla spesa pubblica produttiva e su di un certo grado di disuguaglianza nella distribuzione del reddito e di polarizzazione nelle preferenze dei cittadini in merito alla politica economica. L'innovazione principale di questo contributo risiede nel processo politico che determina la tassazione del capitale, processo basato su "un'attività di influenza" esercitata dalla minoritaria classe capitalista al fine di catturare parte del potere politico a spese della maggioranza dei cittadini. In particolare, gli investimenti in attività di lobbying permettono ai capitalisti di ottenere un via via più basso livello di tassazione del capitale, a beneficio di loro stessi e della crescita economica. L'accumulazione del capitale porta altresì ad un aumento dell'attività di lobbying, al fine di convincere il governo ad implementare un livello di tassazione del capitale sempre più basso, e sempre più affine alle preferenze dei capitalisti. In conclusione, si dimostra la piena convergenza, nel lungo periodo, ad una realtà politica totalmente dominata dal potere di fatto di pochi capitalisti, cioè ad una realtà politico-economica simile ad una "oligarchia tecnocratica" o "plutocrazia".

A hundred men acting uniformly in concert, with a common understanding, will triumph over a thousand men, who are not in accord and can therefore be dealt with one by one. Meanwhile it will be easier for the former to act in concert and have a mutual understanding simply because they are a hundred and not a thousand. It follows, that the larger the political community, the smaller the will the proportion of the governing minority to the governing majority will be, the more difficult will it be for the majority to organize for reaction against the minority.

Gaetano Mosca (1939)

One conclusion is already quite clear, however: it is an illusion to think that something about the nature of modern growth or the laws of the market economy ensures that inequality of wealth will decrease and harmonious stability will be achieved.

Thomas Piketty (2014)

The major contributions who fund political campaign are, by definition, rich (poor people cannot afford to do so), and they are not interested in throwing money away. To believe that the rich do not use their money to buy influence and promote policies they like is not simply to be naïve. Such a stance contradicts the basic principles of economics as well as the ways in which the rich people have amassed their wealth---surely not throwing it around it while expecting no return on it.

Branko Milanovic (2016)

1. Introduction

There is now a wide consensus about the political institutions being of paramount importance for both political and economic outcomes. Formal models indicate that changes in political institutions should be expected to have important consequences in several decision patterns, ranging from income taxation and public good provisions, to the rights of recently enfranchised minorities. Income inequality often importantly interacts with significant changes of the macro-political environment[1]. The available empirical evidence is mixed, though. For example, Rodrik (1999) empirically demonstrates, that democracies do pay higher wages, compared to non-democracies. However, Acemoglu *et al.* (2011), demonstrate that fiscal redistribution in a democracy can sometimes be relatively low, even in presence of high inequality, when the government is controlled by an anti-redistribution coalition involving the rich and the bureaucrats. Such coalition supports the creation of a state's organizational apparatus with weak fiscal capacity, but generating rents for itself. Also, in an important paper, Perotti (1996) casts doubt on the empirical relevance of the Meltzer and Richard's (1981) celebrated positive theory of redistribution, that has been (and still is) an important benchmark in political economy.

More recently, Acemoglu *et al.* (2019) show, that democracies, despite their heterogeneity, tend to outperform on average non-democracies in terms of economic growth, because of their superior capability of creating a playing field for entrepreneurship. However, Barro (1997, 1999), and Mulligan *et al.* (2004) find no substantial variation in policy outcomes and performances between different political regimes, a finding that casts doubt on the economic importance of political institutions.

The broad picture emerging from some of these studies is, thus, that democracies and non-democracies may not be so different in some important dimensions, including the workings of some of their economic institutions (e.g. the structure of their labor market), the programs about income redistribution and public goods provision implemented by the state, and, ultimately, their economic growth rate.

Acemoglu and Robinson (2006, 2008a and 2008b) attempt to reconcile some of these puzzling findings, by observing that political regimes, including democracy, are characterized by a

[1] See Bertola *et al.* (2014) for a rigorous and rather comprehensive discussion of income distribution's role in macroeconomics and in political economy.

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potential de-coupling between the *de jure* and the *de facto* political power. The *de jure* political power is determined by the existing formal political institutions; the *de facto* political power is instead acquired with material means by a small subset of the citizenry, that can afford them. The de-coupling of these two forms of power can lead to the potential capturing of a formally democratic political realm by a population′s (usually affluent) minority. Therefore, while a captured democracy possesses formal political institutions, somewhat similarly to a non-captured or constitutional democracy[2], we opt to refer to captured democracy as a realm, where the *de jure* power is significantly overwhelmed by the *de facto* power. And its pseudo-democratic economic institutions and other economic outcomes may turn out to be both highly distorted and very different from those quite naturally expected in an environment, where the *de jure* power would by far and large prevail; that is, in a constitutional democracy.

Understanding the political logic of captured democracies requires bearing in mind, that formal political institutions are usually defined by Constitutions[3], that carefully allocate different forms of *de jure* political power to different state′s bodies and branches. They therefore also create a checks and balances' system, preventing an excessive concentration of power in any particular articulation of the state. Separation of powers, in turn, credibly ensures the existence of a level playing field for fair economic competition, and represents the political foundation of sound economic institutions. This is the case to a much lesser degree in many non-democracies, as well as in captured democracies. Hereby the endogenously limited importance of the *de jure* power, combined with the predominant importance of the *de facto* political power[4], creates a highly biased environment for economic and political activity, that causes the emergence and persistence of potentially highly distorted and dysfunctional economic institutions (e.g. monopolist output markets, and monopsonistic labor markets, in absence of appropriate regulation). Or, else, it may cause the emergence of very conservative social contracts, especially in democracies, that are either captured or feature some formal political institutions strongly biased in favor of the rich (e.g. Bénabou, 2000). Such arrangements feature very limited redistribution. This may or may not be harmful for economic efficiency and growth per sé, but

[2] The use of the term constitutional democracy, as opposed to captured democracy, is not entirely appropriate, since the latter also usually relies on a Constitution. But such Constitutions' working tends to be highly distorted, or totally subverted, by major investments in the *de facto* political power by the rich elite.
[3] The Constitution can also include highly consolidated political practises. Though not part of any written document, such practices and related traditions are a potential source of *de jure* political power.
[4] Yet, perhaps surprisingly, some form of Constitutionalism exists also in dictatorships. See Ginsburg and Simpser (2013) on this interesting but probably under-researched topic.

tends to increase inequality and, therefore, may potentially undermine the stability of democracy itself in the long run (e.g. Acemoglu and Robinson, 2005)[5].

In this paper we study, how the economic growth process typical of a constitutional democracy[6], can be affected by investments' influence activities by the rich elite. It is assumed, that this democracy features a highly polarized society, divided between a large mass of (relatively poor) workers and a small minority of (very rich) capitalists.

The *de jure* political institutions are basic: people decide by majority voting according the one man-one vote principle, and, since all the relevant standard assumptions apply, a government of the median voter is elected[7]. We don't model in detail the workings of representative democratic government obtaining power and, in particular, the preferences and behavior of politicians in office. Rather, we make the simplifying assumption, that the tax policy preferred by the median voter can be distorted downwards by the rich elite investing in lobbying activities, according to an influence function. Such function intuitively features relatively standard properties, including decreasing marginal returns to the volume of influence activities. Lobbying spending will affect and track the accumulation of capital, and it will gradually turn the original constitutional democracy, into a regime where power is fully captured by the capitalist elite, resembling a quasi-capitalist plutocracy[8].

The economic environment is a simple generalization of Barro (1990)[9], and especially of Alesina

[5] Bénabou (2000) shows that, depending on the balance between distortions and efficiency gains both caused by redistribution, the American social contract, featuring high inequality and low redistribution, may or may not conduct to more economic growth compared to the European social contract, showing opposite traits.

[6] We are agnostic regarding the origin of the *status quo* constitutional democracy. It may have been in place for a short time (and therefore represents a newly created realm), following a political transition from some sort of non-democratic regime previously in place. Or, else, it may have been in power for a while, but experiencing, for some reason, some relative stagnation. Such inertial situation ended with the start of the process, that eventually set in motion the development of the economy. But also, at the same time, triggered the class struggle, that induced the rich elite to acquire *de facto* power.

[7] For instance, electoral competition (hereby not explicitly modelled) may involve two Downsian political parties, solely concerned with winning office, and both committing to implement the same most popular tax policy, if elected. It therefore doesn't matter, which one of the two parties will be actually elected. None of the two parties has any *ex post* commitment problems due to their policy agnosticism. See also Alesina and Rosenthal (1995) for a discussion of political competition with and without policy motivated politicians.

[8] We use the term quasi because the fiscal instrument chosen by the government to raise taxes (proportional taxation) is not necessarily the one preferred by the capitalists.

[9] It is worth noticing that Barro (1990) continued the endogenous growth revolution started by Romer (1986 and 1990). A key result of these papers is to let the marginal productivity of capital remain strictly bounded from below by the rate of time preference. This prevents the economy to fall into the typical neoclassical steady state. Barro obtains this result with following clever and elegant assumption: the aggregate production function depends on productive public expenditures, that potentially create a strong economic role for the state.

and Rodrik (1994), featuring the potential provision of a productive public good. This provision is financed with proportional taxation of capital, allowing growth to be potentially endogenous. The small capitalist elite is in favor of some positive, but relatively limited, capital taxation, maximizing at one time the economy's growth rate and their own welfare. The large mass of workers, instead, in order to redistribute some factor income in their favor, supports a much higher taxation, higher not as high as to shoot down grow. This is essentially in line with Meltzer and Richard's canonical positive theory of redistribution in a democracy. According to this theory, the workers have all the *de jure* political power in a pure constitutional democracy, based on majority voting; therefore, in absence of some form of activation by the rich minority, they would impose their own preferred tax rate.

In our model, instead, we allow the rich to invest in influence activities, or lobbying, on the democratic government, in order to acquire some *de facto* political power, and to tilt fiscal policy in their favor. Following the spirit of a long tradition in the social sciences (including Mosca, 1939; Olson, 1965; Becker, 1983), we assume that only the capitalists, are able to solve the canonical collective action problem (and related crucial *free rider* issue faced by any social group) because of their very small number. Therefore, they are able to get organized and form a pressure group of their own. Nevertheless, coordination is only partial, and each capitalist takes as given the amount of resources invested in political influence activities by their peers.

Because ours is fundamentally a political growth theory, we are chiefly interested in understanding how the process of (endogenous) growth and the de-coupling between different forms of political power affect each other; and, ultimately, how their interaction shapes the economy's dynamic performance, both in terms of development and of income distribution patterns. Our main result is the following: in a model of endogenous growth based on private capital accumulation, and relying both on a non-accumulable factor of production and on a productive public good, *growth stimulates lobbying activities, and vice versa*, in the guise of a two ways interaction[10]. Economic growth and lobbying by the rich elite are linked by the degree of economic polarization present in the society. In a richer and more unequal country, the relatively few capitalists have potentially much more to lose to the workers, if these

[10] As explained in greater detail below, this result is, however, doesn't apply uniformly across different classes of growth models. For example, in Schumpeterian growth models à la Aghion and Howitt (see Aghion and Howitt, 1998, or Acemoglu, 2009, for an introduction to the Schumpeterian framework), lobbying may be harmful for growth. It may be specifically so by slowing down the process of creative destruction. See also the discusssion of this point further below.

politically prevail; therefore, they invest more in the *de facto* power, to prevent their expropriation by the lower classes as much as possible. At the same time, the relatively low taxes induced by capitalists' influence activities on the government stimulate, by raising the interest rate, capital accumulation at the expense of the equalization of the *disposable* factor income distribution[11]. This pattern features a potential complementarity between factor-income inequality and redistributive fiscal policy. In addition, it is broadly consistent with the evidence presented by Barro (2000), showing that inequality encourages growth in developed countries. Forbes (2000) also re-assesses the relation between inequality and growth, using a methodology based on panel estimation; this allows to control for time-invariant country-specific effects, eliminating a potential source of omitted-variable bias. As Forbes writes (2000, p. 869),

"Results suggest that in the short and medium term, an increase in a country's level of income inequality has a significant positive relationship with subsequent economic growth".

Interestingly, the model[12] reaches its balanced growth path by firstly going through a transitional dynamic, where the variables grow at different and time-varying rates. Taxes, in particular, are not constant, but *decrease* over time (together with the implicit redistribution in favor of the workers, that they generate), until they reach the technocratic rate preferred by the rich elite. This tax rat is attained in the quasi-plutocratic balanced growth path. As a result, the post-tax distribution of income becomes more polarized. This implies that the total income accruing to the capitalists constantly increases as a share of the *total* post-tax income.

In addition, the model can help shedding light on the important phenomenon known as raise of fiscal conservatism (e.g. Saint-Paul, 2001). By this expression we mean a progressive retrenchment of the welfare state[13]. This phenomenon has been observed with variable degree of intensity in the last decades in many industrialized democracies, mostly in the U.S. and in the U.K., less so in Continental Europe. This process has been explained with variable degree of

[11] Evidence broadly consistent with this pattern is provided by Acemoglu *et al.* (2015); they argue that the expectation about democracy leading to a reduction of income inequality is not met, when power is captured by a rich elite. Furthermore, the transition to democracy is not necessarily associated to a uniform reduction of inequality, but can lead to changes in patterns of public spending, fiscal redistribution and economic structures, all of them with potentially have ambiguous effects on inequality (e.g. the Jim Crow laws implemented in the post-Civil War Southern U.S. States).
[12] Unlike Bertola's (1993), and Alesina and Rodrik's (1994).
[13] Including, possibly, a reduction of the labor share, (e.g. Karabarbounis and Neiman, 2014).

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success by other prominent theories[14]. Our model potentially helps explaining both: the same broad batter, i.e. the generalized rise of fiscal conservatism, and crisis of the traditional, mid XX century, form of welfare state; as well as the more recent rise of the top 1%, i.e. the unparalleled (through the whole last century) rise of economic fortunes enjoyed by the richest 1% of the population, in the last few decades. Our model emphasizes the interaction of economics and politics in a relatively developed and unequal society, and in particular the greater incentives faced by the rich elite to invest in the *de facto* political power. Additional evidence consistent with this claim has been recently provided by Aghion *et al.*'s book (2021, and especially, Ch. 5)[15], documenting an especially striking fact: the share of national income accruing to the top 1% of the population has increased significantly with the intensity of lobbying between 1998 and 2008. They conclude (2021, Ch. 5, p. 89) that: …] This outcome confirms that lobbying is indeed an other source, distinct from innovation, of inequality at the top. In their Schumpeterian framework, lobbying indeed enables incumbent firms to maintain their market power and their rents, to shield their sector from competition, but also allows them to have easier access to credit and to pay less taxes.

It should be noticed, however, that in a Schumpeterian framework lobbying is likely to be harmful for growth for at least two reasons (Aghion *et al.*, 2021, Ch. 5, p. 92). Firstly: firms destine resources to lobbying at the expense of innovation. Secondly: lobbying slows down the process of creative destruction, that is the essence of growth in any Schumpeterian framework[16]. An excessive political empowerment of the rich may also be harmful for growth in models featuring the scope for efficient redistribution in favor of the poor (e.g. Bénabou, 2000). For example, by allowing them to partially overcome market failures such as credit market imperfections, tending to inhibit their investments in human capital, as well as the acquisition of insurance against idiosyncratic labor income shocks. We therefore remark that our result, i.e. lobbying unambiguously stimulates growth by reducing capital income taxes and thereby stimulating capital accumulation, helps us isolating one potentially important effect of (capitalist) influence activities, but this result cannot be regarded as general, since it hinges on

[14] Notable examples include skilled-biased technical change (e.g. Autor *et al.*, 1998; Acemoglu and Restrepo, 2020) and trade with developing countries (e.g. Autor *et al.*, 2014; Adão *et al.*, 2022). All these explanations posit, for different reasons, a sharp reduction of the demand of unskilled labor. Furthermore, the price, commanded by it in a competitive labor market, falls in relative and *absolute* terms. This fact potentially explains the observed raising inequality patterns usually noticed within many developed countries.

[15] On the relation between inequality and lobbying, see also Aghion *et al.* (2019).

[16] See also Akcigit *et al.* (2023) on this topic.

assumptions that are too model-specific[17].

It is also interesting to observe that in our framework, the overall allocation of political power changes quite substantially, even though the model features no abrupt change in political institutions, triggered for example by revolution, a military coup or a civil war. This occurs as the endogenous relative political weight of the two classes changes considerably over time, in favor of the small minority of capitalists, and in parallel with capital accumulation.

The (pro-rich) peaceful revolution, going on during the growth process, implies that the nature of the political regime also evolves accordingly, from the initial constitutional democracy to an oligarchic technocracy. The constitutional democracy features very little investment in the *de facto* political power by the elite, and is ruled by the median voter, whereas the regime eventually emerging at relatively high levels of economic development, is mostly or entirely controlled by the capitalist minority, and is therefore also referred to a capitalist plutocracy.

Our paper is related to a number of bodies of literatures, stressing the importance of the interaction between inequality and democratic politics in various guises. Firstly, a relatively large set of contributions appeared though the 1990s, and emphasized the complex links existing between economic growth, politics and the distribution of income in non-representative agent setups, such as Bertola (1993, 1996), Perotti (1993), Saint-Paul and Verdier (1993, 1996), Alesina and Rodrik (1994), Persson and Tabellini (1994), Bénabou (1996, 2000), Bourguignon and Verdier (2000). The more recent, influential work of Piketty (2014)[18], is also clearly related to this paper. Piketty argues that an exceptional concentration of economic and political power in a small elite (the top 1%) has been generally observed in the last few decades in the most important economies of the world, as an almost natural consequence of capitalist development[19]. The already mentioned growing importance on money invested by the rich and super-rich in electoral politics, especially in the US, and particularly at the Federal government level (see for instance the studies of Bartels, 2010; Gilens 2012; Gilens and Page, 2014; and Page and Gilens, 2020), is indeed staggering. Gilens (2012) shows, in particular, that the legislators'

[17] The paper's concluding Section provides an additional discussion of this point. It also briefly mentions potential extensions of our basic framework, that may lead to a more general characterization of the process of political lobbying and of its effects on economic growth.
[18] See also Boushey *et al.* (2017), for an extensive critical discussion of Piketty (2014).
[19] Our paper, however, does not hinge on Piketty's famous $r > g$ condition in order to explain the surprising political-economic dynamics, that he documents in his work. Indeed, in our setup r and g are jointly endogenously determined variables, rather than separate elements, as they are in Piketty's (2014) model.

responsiveness to people in the 90th percentile of the income distribution smoothly increases as the issue becomes more relevant to the rich elite. This sharply contrasts with legislators' responsiveness to the issues of concern for the poor (i.e. the bottom 10th percentile of income's distribution) and the middle class (the 50% percentile of income's distribution): this is essentially a flat line, indicating an almost total lack of concern for more salient the issues of the lower and intermediate social classes. Furthermore, as pro-rich policies increase the income of rich, the rich are almost alone in making relevant contributions to politicians, and therefore obtain disproportionate attention from them. Ultimately, the one-person-one vote system is replaced by the one-dollar-one vote rule, which is nothing else that the projection on the political plane of the existing distribution of income, (see Milanovic, 2016, p. 190). Finally, in a very recent contribution, Page and Gilens (2020, Ch. 4, p. 114) add that: As best as we can tell from their contributions to political candidates, most American billionaires tend to be conservative on economic issues. Most of them favor limited social spending, relatively low taxes on upper-income people, and only modest (if any) government regulation of the economy.

Furthermore, we must mention the early 2000s literature regarding the persistence of institutions in presence of reallocation of the $de \quad jure$ political power. These works include the seminal papers of Acemoglu and Robinson (2006, 2008a and 2008b), showing that drastic changes in political institutions don't need to take place along radical transformations in economic institutions. This is, essentially, because in equilibrium the economic elite buy enough of the $de \quad facto$ power to offset the negative (for them) shock to the augmentation of the $de \quad jure$ power of the masses, triggered by a transition to formal democracy. Acemoglu *et al.* (2011) expand on this topic by showing, that the strategic creation by the elite of a state apparatus with limited (inefficiently low) fiscal capacity leads to under-provisioning of public goods.
But this allows the rich elite to preserve much of its power, even after a major political transition[20], by forming a pro $status \quad quo$ perverse coalition with the state's bureaucrats. None of the papers mentioned on the persistence of power across changing political institutions, though, addresses the question of $how \quad economic \quad development \quad interacts \quad with \quad the \quad dynamics \quad of \quad the \quad de \quad jure \quad and \quad the \quad de \quad facto \quad political \quad power, and \quad the \quad related \quad social - economic \quad struggle, within \quad a \quad full - fledged \quad model \quad of \quad endogenous \quad growth, \quad as \quad we \quad do$

[20] The paper in question also contributes to the growing literature explaining how state underdevelopment and failure is connected to the elite's aspiration to both preserve some power (mainly $de \quad facto$), after a transition to formal democracy has occurred, and to the persistence of relatively high inequality (due to low redistribution) in the post-democratic transition period.

hereby.

Also worth to be mentioned, is the literature on lobbying theory, beginning with Becker's classic paper (1983), and including the later contributions of Austen-Smith (1987), Baron (1994), Besley and Coate (2001), and Grossman and Helpman (2002). Relative to all of these important papers, our model relies on a much simpler lobbying process, that is nevertheless applied to the dynamic environment of an infinite-horizon economy growing endogenously, in presence of a fundamental political-institutional conflict.

2. THE MODEL: FOUNDATIONS

We consider a model of endogenous growth, that is partially similar to the one presented in Alesina and Rodrik's (1994) seminal paper on inequality and growth. There is an infinite-horizon economy in continuous time, populated by a finite number of individuals, with identical preferences represented by

$$\int_0^\infty e^{-\rho t} \, ln(c_t^i)dt, \qquad (1)$$

where ρ represents the time discount factor and c_t^i is the consumption at time t of a generic individual i.

Firms operate with an "extended" neoclassical production function. As in Barro's (1990) paper, the technology available in our model relies *inter alia* on the provision of a productive public good by the government. Specifically, the aggregate production function has the following form

$$y = Ak^\alpha g^{1-\alpha} \ell^{1-\alpha}, \qquad (2)$$

with $\alpha \in (0,1)$. In this expression, k stands for the accumulable factor of production, including physical, but also human, capital[21]; g indicates the stock of productive public spending supplied by the government; finally ℓ stands for the total supply of non-accumulable factor of

[21] Since the notion of capital must be broadly interpreted, the model will potentially account for the fact that part of the top earners (i.e. bankers, top managers) will have themselves a dual role of capitalists as well as workers. This fact makes contemporary globalization capitalism partly different from the XIX century patrimonial capitalism, or classical or the Belle Époque capitalism. The latter form of capitalism featured a very high correlation between ownership of capital and high incomes, and was thereby largely dominated by pure reinters only (Milanovic, 2014, p. 527-528).

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production, that is to say, raw labor[22].

Productive public expenditures are financed with proportional capital's taxation, and the government budget constraint is assumed to be always balanced, so that, at each time, we have that the following equation holds[23]

$$g = \tau k. \tag{3}$$

Combining the last two equations, one gets a new form of the production function, namely

$$y = A\tau^{1-\alpha}\ell^{1-\alpha}k. \tag{4}$$

The crucial feature of this last equation is to be linear in the accumulable factor of production, so that, in principle, it can potentially allow the (net) interest rate and marginal productivity of capital not to fall below the rate of time preference (at least if taxes do not increase too much)[24].

We follow Alesina and Rodrik (1994) in considering a generalized version of Barro's model, where the representative agent setup is replaced by the assumption, that people are indeed heterogeneous, in the sense of having a different initial (i.e. at time $t = 0$) relative endowment of capital and labor income. Specifically, citizens differ in their initial *relative* ownership share of the aggregate raw labor stock vs. their relative ownership share of the aggregate capital stock; therefore, for a generic individual i, the following formula applies

$$\sigma_0^i = \frac{\ell^i/1}{k_0^i/k_0} \in [0, \infty], \tag{5}$$

a formula naturally assuming the normalization to 1 of the aggregate stock of unskilled labor. The parameter σ may shape individual's preferences on the tax rate τ, that generates some factor-income redistribution. Therefore, such preferences will depend on the relative

[22] We remark that, unlike private capital, the productive public good g isn't a model's state-variable, but a control variable, linked with taxes and the government (static) budget constraint. See equation (balanced budget) reported below. Futagami *et al.* (1993) consider instead an interesting version of Barro's (1990) model, where g corresponds to a state-variable (public capital), and find that, unlike in Barro's (1990), growth maximization is not equivalent to the maximization of the welfare of the representative agent.
[23] Note that, even if taxes are proportional, more capital-rich individuals contribute more, for any given tax rate, simply because they have more to give. Furthermore, the tax policy in question tends, *inter alia*, to redistribute factor-income from capital to labor by boosting wages (for given capital stock), as demonstrated below.
[24] If, instead, the opposite event happens, growth is well-known to end, due to the evaporation of the individual incentives to save, and the economy ends up in a stationary state.

endowment of non accumulable vs. accumulable factors of production. Note that, while the numerator of formula (5) is, obviously, always constant, the denominator may, in principle, change over time, with individual i potentially getting richer or poorer in terms of relative endowment of capital income[25]. In our framework, though, the initial distribution of income takes a particularly simple form, as we assume the existence of two "social classes" only. A small minority, of finite size λ^k, of the population is formed by capitalists[26], owning *in equal proportion* all the capital, and doing all the savings. They have initially no political power whatsoever, but they alone invest in lobbying activities over time (see below), gaining *de facto* some political voice at the expense of the workers. The set of capitalists is denoted as K ; all capitalists are alike, and therefore (in the symmetric equilibrium we shall focus on), behave like a single "representative capitalist", denoted as i . A large, but of finite size λ^ℓ , mass of individuals is formed by workers, who have no capital and *do not save*, in line with the classic Kaldor-Pasinetti assumption. Workers also do not engage in any lobbying activity, since, as already mentioned, their class is too large to get organized and solve the relevant free-rider problem[27]. They therefore only work, supplying *individually* a quantity of labor equal to $1/\lambda^\ell$ (and therefore a *total* quantity of labor equal to 1 , as mentioned above), and consume entirely their wage income at each point in time. The set of workers is denoted by L . We assume that the total number of capitalists and workers has a size normalized to 1 , i.e. it is the case that $\lambda^k + \lambda^\ell = 1$. It follows from this normalization that equation (2) represents as the same time the aggregate and the *per-capita* production function.

Notice, that the somewhat extreme assumptions made on the initial value of the distributional parameter σ , imply that $\sigma_0 \in \{0, \infty\}$. Each worker has no capital income and therefore $\sigma_0^i = \infty$, $\forall \ i \in L$; each capitalist has no labor income, and therefore $\sigma_0^i = 0$, $\forall \ i \in K$. Because workers don't save by assumption, and capitalists don't have (and never acquire) any labor income, also by assumption, σ_0^i remains constant over time, for any $i \in L \cup K$. The existence of only two

[25] The output of formula (5) is a *datum* of history, reflecting the initial conditions of the economy, that could be any. However, in principle, it may be that σ_t^i becomes different, as times goes by, from σ_0^i , for some t. As explained later, however, this will never occur in equilibrium.
[26] The capitalists elite may be thought of as representing, in particular, the so-called top 1% of the distribution of income in society, when $\lambda^k \downarrow 0$. The assumption is consistent with the observation that in most industrialized nations the distribution of capital income has been extremely unequal, at least over the last fifty years or so. In particular, the corresponding Gini coefficient for capital income has been often around 90% in most industrial countries, since the early 1980s. On the contrary, the corresponding Gini coefficient for labor income has been remarkably lower, by a factor of 50% or so (see Milanovic, 2023, Ch. 7, pp. 272-273).
[27] See the already quoted seminal works of Mosca (1939), Olson (1965), and Becker (1983) on this point.

types of individuals at each point in time implies that, as discussed in greater detail below, only two tax rates are always preferred by the two subsets of *citizens* (L and K), over any other potential arrangement. However, because of lobbying, the political process will generally deliver, along the transition to the balanced growth path, a compromise taxation (varying over time), as also explained below.

Before proceeding, it will be useful to describe the economic environment, beginning with the computation of the factor rental rates (capital and labor) faced by the individuals as a function of the taxes. The individuals act as price-takers in competitive markets[28]. Using the Cobb-Douglas specification, assumed for the production function (and omitting here for simplicity all time subscripts), we have that the post-tax gross and net rental rate of capital read, respectively,

$$r \equiv \frac{\partial y}{\partial k} = \alpha A \tau^{1-\alpha} \equiv r\overbrace{(\tau)}^{+} \text{ , and } r^k(\tau) = [r(\tau) - \tau]. \quad (6)$$

In addition, the post-tax rental rate of labor reads

$$w \equiv \frac{\partial y}{\partial \ell} = (1-\alpha) A \tau^{1-\alpha} k \equiv \omega\overbrace{(\tau)}^{+} k, \text{ and } r^{\ell}(\tau) = \omega(\tau) k. \quad (7)$$

Both formulas (reflecting the normalization to 1 of the aggregate labor supply) obviously apply, since both factor markets are perfectly competitive, and the neoclassical functional theory of income distribution is thus relevant in this setup; therefore, each factor obtains a gross reward equal to its marginal productivity. It is worth to remember, that only capital is taxed, at rate τ, so that its *net* marginal reward is *not* equal to $r(\tau)$ but to $[r(\tau) - \tau]$, and it will turn out to be a non-monotonic function of taxes. The wage rate, instead, increases monotonically with τ (for any given accumulated k). Intuitively, this is the reason, why capitalists will prefer *less* taxation than "workers": they better internalize its cost, including the potentially harmful consequences of too much taxation on economic growth, as well as on their own welfare[29]. It is appropriate to

[28]It is worth to remind that, since the neoclassical theory of income distribution obviously applies, the total factor income accruing to an agent from any factor of production, is simply equal to the marginal productivity of that factor of production, times its personal endowment of that same factor. Also, because of Euler's theorem, all output is exhausted by rewarding all the factors of production (except the public good), that are priced according to their marginal productivity (i.e. there is no left-over income to deal with).

[29] Notice that, while taxes are in principle unrestricted (i.e. they can potentially go all the way up to 100%), equation (net and gross interest rate given tau) makes clear that, in concrete, this is not the case. In particular, the interest rate can't be negative, of course (otherwise nobody would hold any capital), and that implies that $\tau \leq (\alpha A)^{1/\alpha} \equiv \tau^*$. This

specify that the overall net income of any capitalist $i = k$ reads, in the symmetric equilibrium that we will consider,

$$y^k = (\alpha A\tau^{1-\alpha} - \tau)\frac{k}{\lambda^k}. \tag{8}$$

This expression clearly reflects that the aggregate capital stock k is evenly split among the λ^k equal capitalists, i.e. the stock of capital owned by a generic capitalist $i \in K$, reads $k^i = \lambda^k \; k$.

In addition, the overall net income of a generic worker ℓ reads

$$y^\ell = \frac{(1-\alpha)A\tau^{1-\alpha}k}{\lambda^\ell}. \tag{9}$$

This expression reflects, that each worker supplies individually $1/\lambda^\ell$ units of labor[30].
Of particular importance, among the menu of feasible taxes that the government can levy, is the (constant) tax rate, defined as τ^k. This specific tax rate maximizes the net interest rate (or net marginal productivity of capital); moreover (as we will demonstrate later), it also maximizes the welfare of the capitalists in a hypothetical oligarchy, where this class is fully in control. Such tax reads

$$\tau^k = [\alpha(1-\alpha)A]^{\frac{1}{\alpha}}. \tag{10}$$

Also important is another (constant) tax rate, τ^ℓ, that maximizes instead the welfare of the (pure) workers, indicated hereby with the superscript ℓ. This tax rate obtains in absence of *any* lobbying activity by the capitalists, namely in a political environment, where workers, who are the absolute majority of the population (including the median voter), are always fully in power. Hereby, therefore, the median voter theorem (henceforth MVT)[31] applies trivially, and the tax τ^ℓ in question is implicitly defined by the equation[32]

equation potentially introduces an endogenous "state capacity" constraint into the model; but it is not so relevant hereby, as it will never bite in practice.

[30] These formulas immediately reveal that the capitalists' income is much higher than the workers' income, as λ^ℓ is assumed to be much larger than λ^k, whereas λ^k may tend to 0 in the limit. Hence, referring to the capitalists as "rich" and to the workers as "poor", is fully justified. Both the functional distribution of income and the very different size of the two classes are the reason of such justification.

[31] See Austen-Smith and Banks (2005) for an excellent introduction to social choice theory, and discussion of the median voter theorem.

[32] Formally, equation (11) emerges as the first order condition of the following problem

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$$\tau^{\ell} - \alpha A(1-\alpha)A\tau^{\ell^{1-\alpha}} = \rho(1-\alpha). \quad (11)$$

Intuitively, at $\tau = \tau^{\ell}$ the marginal gain of increasing taxation, in terms of boosting the current wage rate, is just offset by the corresponding marginal loss. This marginal loss consists in the reduction of the net interest rate, and therefore of the growth rate of the aggregate capital stock, and of future wages. Importantly, τ^{ℓ} represents the highest level of redistributive taxation supported by the political-economic system, as it reflects the pure fiscal policy preferences of the virtual median voter (who owns no capital); relatively to such benchmark, tax policy can only be distorted downwards, by the political pressure exercised by the capitalist elite on the government[33].

$$\max_{\tau} \int_0^{\infty} e^{-\rho t} \ln(c_t^i)dt = \int_0^{\infty} e^{-\rho t}[(\alpha A\tau^{1-\alpha} - \tau - \rho)t + \ln((1-\alpha)A\tau^{1-\alpha}k_0)]dt$$

$$= \frac{\alpha A\tau^{1-\alpha} - \tau - \rho}{\rho^2} + \frac{\ln\left((1-\alpha)A\tau^{1-\alpha}k_0\right)}{\rho}.$$

An expression obviously reflecting that

$$\ln c_t^i = \ln((1-\alpha)A\tau^{1-\alpha}k_0 e^{(\alpha A\tau^{1-\alpha}-\tau-\rho)t}).$$

It is straightforward to verify, that the first order condition for the last equation corresponding to the integral, with respect to τ, or

$$\tau[1 - (\alpha(1-\alpha)A)\tau^{-\alpha}] = \rho(1-\alpha).$$

is equivalent to equation (11) with $\tau = \tau^{\ell}$. It can be verified, in addition, that the relevant second order condition for a maximum point is satisfied.

[33] Equation (11) represents the special case of a more general equation reported in Alesina and Rodrik (1994, equation (15), p. 474), obtaining for $\sigma_0^i = \infty$, and defining implicitly the preferred tax policy of the generic individual i with $\sigma^i \in [0,\infty)$. The equation in Alesina and Rodrik's reads

$$\tau^i\left[1 - \alpha A(1-\alpha)\tau^{i^{-\alpha}}\right] = \theta^i(\tau^i)\rho(1-\alpha),$$

with

$$\theta^i(\tau^i) \equiv \frac{\omega(\tau^i)\sigma^i}{\omega(\tau^i)\sigma^i + \rho}.$$

It can be demonstrated that, in agreement with Meltzer and Richard's (1981), the tax rate τ^i increases with the distance between the income of the mean and of the median voter, when i represents the median voter.

Our equation (11) reported in the main text can be regarded as a special case of the Alesina and Rodrik's (1994) equation reported above, obtaining for $\sigma^i \to \infty$ (i.e. workers have no capital income's endowment at all). Notice that, in this case $\theta^i(\tau^i)$ converges to 1 for any τ^i. Interestingly, it can be demonstrated, that the preferred tax rate of a pure worker (or the tax rate implemented by a “left-wing populist” government), also leads to positive long run growth. Both this specific voter and its own government rationally understand that: wages (like gross interest rates) depend positively on taxes, but wages (unlike gross and net interest rates) also depend positively on capital. Therefore, a pure worker uses taxes to both boost its own static wage income, and to promote capital accumulation, in order to increase its future path of labor income, depending on the future path of k. This is also the reason why expropriating entirely the capitalists, a policy that would obviously stop growth altogether, is not a desirable policy, even for people owning no capital whatsoever.

2.1 Political Process and Lobbying Technology

As anticipated in the Introduction, we consider as basic political framework a democracy originally based on the $de \quad jure$ power only, but eventually turning into a captured democracy, due to the ongoing growth of the $de \quad facto$ power of the rich elite. Therefore, the political process is only partially based on majority voting, and the government in office, hereby not explicitly modelled, imperfectly represents a virtual median voter. Democracy potentially evolves according to the lobbying activities performed by the rich elite on the government (tending to endogenously increase over time), shifting the balance of overall power towards the latter class. The rich elite then gradually acquire more $de \quad facto$ political power, whereas formal institutions don't change.

The representative government sets the capital tax rate τ , and fiscal revenues are used to finance the provision of the productive public good g[34]. Such policy has different effects: first and foremost, it allows the economy to grow endogenously, by making the production function linear in k (see equation (4)). In addition, it affects in a non-trivial way the functional distribution of income and factor shares: the rate of reward of capital income may increase or decrease with it (since it is "humped shaped" in the tax rate), whereas the wage rate is always increasing in τ , for any given k . These different effects generate a fundamental distributive conflict between capitalists and workers. Just as in Alesina and Rodrik (1994), the former would like taxes to be set at the level just maximizing the economic growth rate (i.e. the net interest rate). Workers instead, would like taxes to be set at a higher level, and are prepared to trade-off some growth with a static expansion of their wage income.

This conflict is resolved by the postulated political process, reflecting an $ad \quad hoc$ generalized democratic decision rule (as opposed to a full-fledged dynamic political game), that gives weight both to the preferences of the mass of workers and to the small rich elite. Crucially, such weight is $endogenous$, and depends on the overall political influence effort exercised by the rich elite.

Specifically, we assume the existence of a "tax function", $\tau(\cdot) \ : R_+ \rightarrow [0,1]$, depending on the total pressure, P_t^K , exercised at each point by the capitalist class (and defined more formally below). This has the following properties: it is a smooth, everywhere strictly decreasing function, and featuring diminishing returns to scale, i.e. $\tau'(\cdot) < 0$ and $\tau''(\cdot) > 0$. In addition, the

[34] This occurs at balanced budget (i.e. there is no public debt); see equation (3).

following "initial condition" and "limit condition" at the boundary of its domain are satisfied,

$$\tau(P_t^K = 0) = \tau^\ell, \tag{12}$$

and

$$\lim_{P_t^K \to \infty} \tau\,(P_t^K) = [\alpha(1-\alpha)A]^{\frac{1}{\alpha}} \equiv \tau^k. \tag{13}$$

Some comments are necessary here to explain the assumptions made above. Because, unlike in Alesina and Rodrik's (1994), we are now in a partially captured democracy, where the median voter theorem, does not apply anymore, together with the one man-one vote principle, underpinning it. Rather, fiscal policy reflects a "compromise" between the ideal policy of the (pure) workers, and of the (pure) capitalists. Workers trivially include the median voter, since they are all identical and make up for more 50% of the "electorate". Capitalists, instead, are exclusively concerned with the maximization of the economy's growth rate, equivalent to the maximization of their own welfare (just as in Alesina and Rodrik's paper). Such compromise reflects the relative political power of the two social classes in question. Crucially, this is *endogenous* due to the potential influence activity on the government exercised by the "capitalist class". In particular, as equation (12) highlights, the political process implements, in absence of any lobbying activity by the capitalists, the very preferred policy of the workers, τ^ℓ. This policy simply corresponds to the *virtual* median voter equilibrium of the two classes-society we are considering. However, as the capitalists keep accumulating wealth, they will also invest more and more in lobbying, in order to reduce the rate of capital taxation. The model flexibly accommodates for the corresponding increment in their *de facto* political power, at the expenses of the *de jure* political power of the workers. In other words, political institutions endogenously change in a peculiar way. While formal political institutions (i.e. democracy) don't change, the real overall political power's allocation features a smooth transition from the one obtaining in a pure, or constitutional, democracy, to the one emerging in a partially captured democracy. Such regime, featuring a time-varying mixture of the *de jure* and the *de facto* political power of the two social classes, will eventually become entirely hegemonized by the rich elite in the very long run, due to the dynamics of their political pressure (that will grow boundlessly).

In particular, in the long run, the political process will implement a rate of capital taxation equal to the very preferred tax rate of the capitalist, τ^k ; i.e. the one maximizing the net marginal productivity of capital (see equation (6)). This policy emerges in what we may term a pure capitalists-dominated technocratic regime. Or, alternatively, an oligarchic technocracy, i.e. a political regime solely concerned with economic growth maximization. In our framework, this equals the maximization of the intertemporal welfare of the pure capitalists. In order words, the captured democracy effectively evolves in the long run into the government of the capitalists only. This occurs, when the capitalists become rich enough to obtain the full control of the political system. We remark that in our setup, the rich do achieve this goal by using their own means only, i.e. money and other financial resources, as opposed to any kind of violent activity (exercised, for example, by forming a coalition with the army or paramilitary troops[35]).

It is convenient to introduce here a $specific \quad form$ for the tax function, that we shall adopt at some point. The functional form assumed entails no substantial loss of the generality of any of the results, considerably simplifying the model's solution[36], and can be written as

$$\tau\left(p_t^i + P_t^{K\setminus\{i\}}\right) = \tau^k + \left(\tau^\ell - \tau^k\right) exp\left(-P_t^{K\setminus\{i\}} - p_t^i\right) = \tau^k + \Delta\, exp\left(-P_t^{K\setminus\{i\}} - p_t^i\right). \tag{14}$$

where p_t^i denotes the lobbying spending of capitalist i at time t , and

$$P_t^{K\setminus\{i\}} \equiv \sum_{j\in K\setminus\{i\}} p_t^j \,, \tag{15}$$

denotes the total pressure exercised at time t by the whole capitalist class, with the $exception$ of the representative capitalist i , that takes the former as $given$ [37]. The $total$ political pressure (already introduced informally above in equation (13)) exercised by the capitalists class as whole at time t , and additive in all of its components, will instead be denoted as

$$P_t^K = \sum_{j\in K} p_t^j \,. \tag{16}$$

[35] On civil-military politics and political transitions see, for example, Acemoglu *et al.* (2010).
[36] The specific form in expression (14) will be useful at some point to control the ratio $\tau''\left(p_t^i\right)/\tau'\left(p_t^i\right)$, thereby avoiding a potentially troublesome form of indetermination in computing an important limit. Any other functional form, achieving the same result, is essentially equivalent, in terms of the model's solution.
[37] Notice, the use of the summation symbol, reflecting that the number of capitalists is finite.

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We will naturally focus on a symmetric equilibrium, where all the (identical) capitalists make at each time the same decisions. Such symmetry assumption implies, in particular, that each and all of λ^k capitalists present in the economy, will choose the same lobbying effort, equal to p_t^i. Therefore we can write that, in equilibrium, we have that

$$P_t^K = p_t^i + P_t^{K\backslash\{i\}} = \lambda^k p_t^i, \quad (17)$$

Henceforth, we will refer to taxation as $\tau(p_t^i)$, rather than as $\tau(P_t^K)$, whenever that causes no confusion, in order to make the notation less heavy.
Equation (14) comprises in addition the term Δ, which is defined as

$$\Delta \equiv \tau^\ell - \tau^k, \quad (18)$$

and reflects the extent of potential policy *polarization*, i.e. the difference between the "ideal" taxes of the workers (τ^ℓ), and of the capitalists (τ^k) respectively[38].

Expression (14) has an interesting interpretation as a weighted average of the preferred tax of the pure capitalists, and of the pure workers. The taxes are endogenously weighted by one factor, $exp\left(-P_t^{K\backslash\{i\}} - p_t^i\right)$, reflecting the endogenous degree of political pressure exercised by the economic elite on the government. So that, as already mentioned above, in absence of any whatsoever lobbying activity by the rich, the MVT applies[39]. As political pressure increases, equilibrium taxes decrease. It is possible to demonstrate that they converge, to the preferred taxes of the rich, once the economy reaches its balanced growth path, as lobbying activity eventually becomes infinite[40].

It is interesting to remark that, according to expression (12), higher policy polarization leads to higher taxation, for any $given$ level of political pressure. This result obtains, because the preferred tax τ^ℓ of the virtual median voter $increases\ \ with\ \ polarization$. On the contrary, the preferred tax rate of the capitalists τ^k depends only on technological parameters. In particular,

[38] We remind that τ^ℓ and τ^k represent, respectively, the tax chosen by the median voter a pure democracy (i.e. in absence of any lobbying by the capitalists), and by the elite in an oligarchy, where the capitalists have full power. Both happens to be constant along the equilibrium path (see Alesina and Rodrik, 1994).
[39] The MVT always applies in the model of a fully consolidated, constitutional democracy, as proposed by Alesina and Rodrik (1994).
[40] However, investment in lobbying stops increasing in balanced growth, and therefore becomes negligible with respect to the growing variables, such as capital and consumption.

it depends only on technological parameters, due to its purely technocratic, growth maximizing nature. Therefore, for any $given$ (finite) p_t^i, polarization unambiguously increases taxation in expression (14)[41]. Nevertheless, higher polarization induces a "defensive reaction" in the rich, namely to lobby more in order to protect their wealth, and this indifferent effect leads to an a $priori$ ambiguous overall impact of polarization on taxes. As a result, the effect on economic growth, is also ambiguous, off the balanced growth path.

It can be verified, that expression (12) satisfies all the assumptions made concerning the tax function $\tau(\cdot)$. In particular, $\tau(\cdot)$ decreases with p^i because (recall that $\tau^\ell > \tau^k$); in addition, $\tau_p(\cdot)$ decreases with p^i, but at an increasing rate, since the following formulas apply[42]

$$\tau_p^{'}\left(p_t^i\right) = -\Delta\, exp\left(-P_t^{K\setminus\{i\}} - p_t^i\right) < 0, \tag{19}$$

and

$$\tau_{pp}^{''}\left(p_t^i\right) = \Delta\, exp\left(-P_t^{K\setminus\{i\}} - p_t^i\right) > 0. \tag{20}$$

3. THE POLITICAL-ECONOMIC OPTIMIZATION PROGRAM OF CAPITALISTS

The program, that capitalist i solves, consists in maximizing its discounted lifetime utility, given both the static constraint, reflecting its present income, and the dynamic constraint, representing the evolution of its wealth. At each point in time, the dynamic reflects its endowment of capital, its consumption decision, its $lobbying\ \ effort$ and the government's policy; taxes now depend both on the fiscal preferences of the mass of workers and the capitalist class, together with capitalists' lobbying activity. As mentioned, the government's policy consists in the tax rate τ_t levied on capital income at time t, in order to finance the provision of the productive public good g_t [43]. Taxes will vary over time, reflecting the potential variation of the

[41] This effect is somewhat in the same spirit of the effect of income inequality, in the potential two classes version of the Meltzer and Richard's (1981) model. There, as it is well known, a higher distance between the income of the mean and of the median voter, i.e. higher income inequality, increases the preferred tax rate of the median voter relative to the preferred tax rate of the mean voter. As such distance increases, the potential scope for fiscal redistribution increases as well.

[42] Notice that in both formulas, which are assumed to apply off-equilibrium, $P_t^{K\setminus\{i\}}$ is taken as given, and therefore does not change with p_t^i. In the symmetric equilibrium we will look at, instead, $P_t^{K\setminus\{i\}}$ will change with p_t^i, as the individual levels of pressure are clearly strategic complements.

[43] However, because taxes depend, $inter\ \ alia$, on capitalists' lobbying, they are not taken as given anymore by the individuals (as they are in Alesina and Rodrik, 1994).

intensity of the lobbying activity[44]. Assuming (recall equation (1)), logarithmic preferences and a discount rate ρ of future welfare, the generic capitalist i solves the following problem

$$\max_{\{c_t^i, p_t^i\}} U_0^i\left(\{c_t^i\}\right) = \int_0^{\infty} e^{-pt}\, ln(\, c_t^i) dt, \tag{21}$$

subject to the static and dynamic budget constraint of the same individual, that read, respectively,

$$y_t^i = [r(\tau_t) - \tau_t] k_t^i. \tag{22}$$

and

$$\dot{k}_t^i \equiv \frac{dk_t^i}{dt} = [r(\tau_t) - \tau_t] k^i - c_t^i - p_t^i. \tag{23}$$

Equation (23) is the differential equation describing the evolution of the capital stock owned by a capitalist. We remind that, all of them are initially equal and do remain equal in the symmetric equilibrium we will focus on, just like the workers; furthermore, all of them own only capital income. Equation's (23) right-hand-side includes income, simply equal to its post tax capital income, net of consumption, and net of the lobbying expenditures p^i incurred to influence government's fiscal policy. Using equation (6), we write equation (23) in the growth rate form

$$\frac{\dot{k}_t^i}{k_t^i} = \left[aA\tau^{1-\alpha}\left(p_t^i\right) - \tau\left(p_t^i\right)\right] - \frac{c_t^i}{k_t^i} - \frac{p_t^i}{k_t^i}. \tag{24}$$

This is one of the main innovations of our model: taxes $are\ \ not$ taken as given anymore and are not perceived to be constant by the individuals, $but\ \ reflect\ \ the\ \ time - varying\ \ lobbying\ \ activity\ \ done\ \ by\ \ the\ \ rich\ \ elite$.

The description of the capitalists' program is completed by the writing of the usual transversality condition, establishing that the shadow value of k_t^i must by asymptotically nil, or

$$\lim_{t\to\infty} \mu_t\, k_t^i = 0.$$

[44] As we shall see, taxes will be constant in the balanced growth path, eventually reached by the economy, after experiencing a process of transitional dynamics. When the economy is off the balanced growth state, consumption and capital grow at a different, and time-changing, rate. Taxes, as already mentioned, are also not constant, and public spending and lobbying (both as a share of capital) aren't.

Finally, we assume that the stock of initial capital is given, and equally distributed among capitalists, i.e. we have that

$$k_0 = \lambda^k k_0^i > 0, \text{ with } k_0^i = k_0^j, \forall\, i \text{ and } j \in \mathrm{K}, \text{ given.}$$

Moving forward, by standard arguments (i.e. Pontryagin's Maximum principle[45]), the capitalists solve their dynamic optimization problem by maximizing the following Hamiltonian function

$$H = e^{-\rho t} ln(c_t^i) + \mu_t\{[aA\tau^{1-\alpha}(p_t^i) - \tau(p_t^i)]k_t^i - c_t^i - p_t^i\}. \tag{25}$$

The standard conditions leading to the maximization of the Hamiltonian function above, include the first order condition for consumption, or

$$\frac{\partial H}{\partial c_t^i} = e^{-\rho t}\frac{1}{c_t^i} - \mu_t = 0, \tag{26}$$

a condition leading to the law of motion of consumption itself, that depends on the dynamics of the Hamiltonian multiplier μ, so that

$$\frac{\dot{c}_t^i}{c_t^i} = -\frac{\dot{\mu}_t}{\mu_t} - \rho. \tag{27}$$

In addition, we have a *novel* first order condition, regarding the new control variable, represented by the intensity of the lobbying activity, and reading

$$\frac{\partial H}{\partial p_t^i} = \mu_t\{[\alpha(1-\alpha)A\tau^{-\alpha}(p_t^i) - 1]\tau_p'(p_t^i)k_t^i - 1\} = 0. \tag{28}$$

Furthermore, the solution of the dynamic program in question requires, that the co-state variable μ satisfies the following differential equation

$$-\dot{\mu}_t = \frac{\partial H}{\partial k^i}, \tag{29}$$

that leads to the following differential equation for the Hamiltonian multiplier μ

$$-\frac{\dot{\mu}_t}{\mu_t} = [aA\tau^{1-\alpha}(p_t^i) - \tau(p_t^i)]. \tag{30}$$

[45] See Liberzon (2012), for an excellent introduction to the calculus of variations and to optimal control theory.

3.1 *Towards the Full Solution of the Capitalists' Dynamic Optimization Problem*

As, obviously, $\mu_t \neq 0$, equation (28) implies that,

$$\left\{\left[\alpha(1-\alpha)A\tau^{-\alpha}\left(p_t^i\right)-1\right]\tau_p^{'}\left(p_t^i\right)\right\}k_t^i = 1. \tag{31}$$

The interpretation of this condition is straightforward: at equilibrium, for a capitalist, the marginal gain from lobbying, in terms of reduction of the fiscal burden on its income (the net interest rate times the stock of capital accumulated), equals to its corresponding marginal cost of lobbying, that equals to 1.

Equation (31) is of utmost importance, since it defines the political pressure schedule $p_t^i = p\left(k_t^i\right)$, as an *implicit function* of the stock of capital k_t^i; p_t^i also depends parametrically on the distance Δ (defined by equation (18)) between the tax rate preferred by the workers and by capitalists[46]. The parameter Δ, as we know, reflects the redistribution potential of a constitutional democracy (where the *de jure* political power alone always matters) vs. a fully captured democracy (where the *de facto* political power alone always matters). We further proceed to characterize some of equation's (31) most important properties. Notice firstly that, by assumption, taxes are decreasing in political pressure, i.e. $\tau_p'(\cdot) < 0$. This fact, and equation (31)

$$\left[\alpha(1-\alpha)A\tau^{-\alpha}\left(p_t^i\right)\right] < 1,$$

imply that, for any $p_t \in R_+$, the following inequality holds

$$\tau\left(p_t^i\right) > [\alpha(1-\alpha)A]^{\frac{1}{\alpha}} \equiv \tau^k. \tag{32}$$

That is, taxation, under any finite level of lobbying, is strictly *greater* than the growth maximizing tax rate τ^k, but it is locally declining, whereas the net interest rate (corresponding to the term in curly brackets in equation (31)) is locally increasing. In other words, lobbying

[46] We remark that equation (31) only applies to an interior solution for the lobbying effort (i.e. $p_t^i > 0$). An interior solution is, indeed, not guaranteed to always exist, as the tax function $\tau(\cdot)$ does not satisfy all Inada's conditions. Nevertheless, it is straightforward to show that an interior solution for political pressure always obtains when k_t^i is above some threshold $\hat{k}$. By definition, at $\hat{k}$ the marginal gain from lobbying at $p_t^i = 0$, equals to the marginal cost (equal to 1). We assume that the initial capital stock k_0 is high enough to guarantee that political pressure is always positive, i.e. that $k_0 > \lambda^k \hat{k}$. We also remark that, if this condition is not satisfied, the economy would experience an initial period of growth, where the capitalists have no *de facto* power at all, and the preferred policy of the median voter τ^ℓ is always implemented by the political process. All of this happens until the capital stock becomes high enough to trigger some positive investment in lobbying, according to equation (31).

helps aligning equilibrium taxes and net interest rates to the ideal fiscal policy of the pure capitalists, but a gap keeps existing, reflecting the (partial) persistence of the *de jure* political power of the workers, as long as the economy does not reach its balanced growth path. When this event happens, instead, it features the complete erosion of any residual formal political power of the lower class, due to the overwhelming political pressures, exercised by the rich capitalists. To such a volume of pressure, the government in office responds by implementing exactly the capitalists's ideal tax policy.

We can now demonstrate two noteworthy results, respectively connecting lobbying with both capital accumulation and the potential redistribution cleavage Δ, allowed for *across* the two extreme political environments featured in our model.

Remark 1 *The level of political pressure exercised by the representative capitalist is a smooth function* $p_t^i = p(\cdot)$ *of its capital stock. The function increases with the representative capitalist's own endowment of capital, i.e.* $p_k'(k_t^i) > 0$ *, and therefore, as economic growth progresses. Also, in the limit, it is the case that* $p(k_t^i) \to \infty$ *, as* $k_t^i \to \infty$ *.*

Proof. See Appendix.

Remark 2 *Political pressure by the representative capitalist also positively depends on* Δ *, i.e. the parameter reflecting the relative redistribution potential in a constitutional democracy vs. a technocracy. Therefore, we have that* $p_t^i = p(\cdot;\Delta)$ *, with* $p_\Delta'(\cdot;\Delta) > 0$ *.*

Proof. See Appendix.

An immediate consequence of Remark 1 is that the overall political pressure exercised by the capitalists increases as the economy grows. Hence, richer economies experience *higher* investments in political influence by the economic elites, that lead to an expansion of the relative political power of the capitalists[47]. A *lower* degree of fiscal redistribution (hereby in the

[47] Incidentally we remark, that economic growth doesn't need to make democratic institutions stronger. This is because, as we have just shown, growth tends to shift the balance between the *de jure* political power of the workers and the *de facto* political power of the capitalists, reducing the relevance of the formal institutions, that are the foundations of a constitutional democracy. This result appears not to square very well with the celebrated modernization hypothesis, (e.g. Lipset, 1959), according to which economic growth leads to the emergence, or to the consolidation of democracy. It is worth mentioning here, that the influential study of Acemoglu *et al.* (2008), is unable to find evidence of a causal relation linking economic growth to democratization over a relatively long period of time.

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form of wage subsidization), is what ultimately follows. However, it should be added that the lower classes also potentially gain from more economic growth, since wages are a linear function of the aggregate capital stock, as we already know[48].

Remark 2 reflects the "defensive" role of lobbying for the capitalists' economic interests: as the scope Δ of potential expropriation of the rich in a constitutional democracy vs. a technocratic oligarchy increases, the capitalists may attempt to defend their wealth by lobbying more. Indeed, it is possible to show, that the $difference$ between the tax preferred by the virtual median voter, τ^{ℓ}, and the rate of overall taxation delivered by the political process, $\tau\left(p_t^i;\Delta\right)$ may increase in Δ. The delivered policy (unlike τ^{ℓ}) depends on the policy polarization parameter Δ both directly and indirectly, as in equilibrium p_t^i is a function of Δ (and $P_t^K = \lambda^k p_t^i$). Letting

$$\tau^{\ell} - \tau\left(\lambda^k p_t^i;\Delta\right) = \Delta - \Delta\, exp\left(-\lambda^k p_t^i\right),$$

be an expression resulting from a straightforward transformation of the tax function (functional form for the tax function), and recalling that $\tau\left(p_t^i;\Delta\right) = \tau\left(\lambda^k p_t^i\left(k_t^i;\Delta\right);\Delta\right)$, we have that

$$\frac{\partial\left[\tau^{\ell} - \tau\left(\lambda^k p_t^i\left(k_t^i;\Delta\right);\Delta\right)\right]}{\partial\Delta} = 1 + \Delta\, exp\left(-\lambda^k p_t^i\right)\lambda^k \frac{\partial p_t^i}{\partial\Delta} > 0. \tag{33}$$

This result represents, in some broad sense, a reversal on Meltzer and Richard's (1981) canonical logic, as the tax rate actually implemented by the political process, decreases $relative$ to the tax rate ideally preferred by the workers (and by the virtual median voter in particular), as the policy polarization parameter Δ increases[49]. The parameter Δ also reflects, in some broad sense, the extent of inequality existing in the society[50].

They interpret this finding by arguing, that the many previous studies on this matter mistakenly failed to control for country fixed effects; these effects might be correlated with both growth and democracy.

[48] In a large class of endogenous growth models (including ours of course), wages are linear in the aggregate capital stock. Therefore, a rapid accumulation of capital (at the expense of a lower wage lower subsidization by means of high capital taxes) is partly beneficial for the workers themselves. As equation (7) shows, the wage rate grows at the same rate of the capital stock (at least at constant taxes, that emerge in balanced growth). Therefore, a faster capital growth rate tends to enrich the workers as well, since capital accumulation by the rich, in some sense "trickles-down" on the poor themselves (see Aghion and Bolton, 1997).

[49] Notice that, as τ^{ℓ} does not depend on Δ, $\tau\left(p_t^i\left(k_t^i;\Delta\right);\Delta\right)$ must necessarily fall. This is to make sure that the left-hand-side of equation (33) is positive, just as its right-hand-side.

[50] It must be borne in mind that the tax rate τ^{ℓ} preferred by the pure workers, corresponds to an $infinite$ value of the statistic $\sigma^m - 1$. Such parameter expresses the distance between the mean and the median income, that represents the indicator of income inequality used in Alesina and Rodrik's (1994). It cannot, obviously, increase beyond the value of infinity, that reflects the maximum possible level of inequality, and characterizes the position of the pure workers in the distribution of income. On the other, the tax rate τ^k preferred by the pure capitalists, depends only on

We can additionally demonstrate an important result concerning the limit behavior of political pressure, as $k_t^i \to \infty$. This result will be specifically useful in the characterization of the model's balanced growth path.

Remark 3 *It is the case, that the following limit result holds,*

$$\lim_{k_t^i \to \infty} \frac{\partial p_t^i(k_t^i)}{\partial k_t^i} = \lim_{k_t^i \to \infty} \frac{\partial p(k_t^i)}{\partial k_t^i} = 0. \tag{34}$$

In addition, we have the straightforward consequence that

$$\lim_{k_t^i \to \infty} \frac{p_t^i(k_t^i)}{k_t^i} = \lim_{k_t^i \to \infty} \frac{\partial p_t^i(k_t^i)}{\partial k_t^i} = 0. \tag{35}$$

Proof. See Appendix.

Combining equation (27) with equation (30), we can obtain at this point the full characterization of the law of motion of the consumption of the representative capitalist, that, taking advantage of Remark 1, reads,

$$\frac{\dot{c}_t^i}{c_t^i} = \left[aA\tau^{1-\alpha}(p_t^i) - \tau(p_t^i)\right] - \rho = \left[aA\tau^{1-\alpha}\left(p(k_t^i)\right) - \tau\left(p(k_t^i)\right)\right] - \rho. \tag{36}$$

Notice that equation (36) reflects the preliminary fact, that taxes are a function of political pressure (see (14)), but political pressure is obviously also endogenous, and determined in the model's dynamic equilibrium. In addition (see equation (24), we have that the dynamics of k_t^i follows the rule

$$\frac{\dot{k}_t^i}{k_t^i} = \left[aA\tau^{1-\alpha}(p_t^i) - \tau(p_t^i)\right] - \frac{c_t^i}{k_t^i} - \frac{p_t^i}{k_t^i} \tag{37}$$

$$= \left[aA\tau^{1-\alpha}\left(p(k_t^i)\right) - \tau\left(p(k_t^i)\right)\right] - \frac{c_t^i}{k_t^i} - \frac{p(k_t^i)}{k_t^i}.$$

At this point, we have characterized the dynamic evolution of capitalists' consumption and

technological parameters, and doesn't depend on any inequality index. Because the coefficient σ does not enter into the definition of Δ, it is not possible to do rigorously any comparative statics with it in our setup.

investment (equations (36) and (37)), as well as the level of lobbying activities performed by each capitalist; furthermore we have characterized the corresponding taxes implemented by the government (as well as the level of productive public good provision) as a function of accumulated capital stock.

In the present setup, however, unlike in Alesina and Rodrik (1994) and other classic models of endogenous growth (e.g. Romer, 1986 and 1990; Barro, 1990), the economy doesn't immediately reach its balanced growth path, but experiences a transitional dynamic, because taxes, as well as lobbying activity, change over time.

To make further progress in the solution of the model, we therefore need to carefully study the behavior of the dynamical system for consumption- and capital-path describing equations (capitalists consumption growth rate final) and (capital growth rate final). Such analysis is performed in the following Section.

Before doing so, a parametric restriction needs to be introduced, in order to make sure, that the net interest rate remains strictly bounded from below by the subjective rate of time preferences, making endogenous growth possible. Because, as we have seen before, the net interest rate depends on time-varying taxes, depending themselves in turn on the capital stock of the economy, we could introduce such crucial restriction only at this juncture of the paper.

Condition 1 *We assume that the following condition is satisfied:*

$$R(k_0) \equiv \alpha A \tau^{1-\alpha}\big(p(k_0)\big) - \tau\big(p(k_0)\big) > \rho. \tag{38}$$

We hereby define Condition 1. This Condition states that, the net marginal productivity of capital strictly exceeds the rate of time preference at time $t = 0$, when $k = k_0$. Therefore, growth is possible initially (i.e. at time $t = 0$) and, due to the smoothness (that is straightforward to prove) of the function $R(\cdot)$, it is also possible for values of k not too far above k_0 as well.

Importantly, the same Condition 38 turns out to hold for any k in the interval $[k_0, \infty)$, i.e. endogenous growth always occurs over this parametric range.

Remark 4 *The function* $R = R(k)$ *, defined in expression (38), and representing the net interest rate, is strictly increasing in* k *over the range* $[k_0, \infty]$ *. It follows that Condition 1 is always satisfied* $\forall\ k \in [k_0, \infty]$ *as well.*

Proof. See Appendix.

3.2 Transitional Dynamics and Model's Balanced Growth Path

Observe that, as reflected in our notation, we have obtained at this point a system of differential equations, describing the simultaneous evolution of consumption and capital, i.e. one of the two control variables, and the model's state variable. Equation (31) defines p_t^i as an implicit function of k_t^i . This means, that the model's solution leads to the equilibrium expression of the capitalist's political pressure, in the form of the function $p_t^i = p(k_t^i)$. This further implies that the pair of differential equations (36) and (37) correspond to a dynamical system in *two* variables, c_t^i and k_t^i .

Because we are dealing with an endogenous growth model, we can't look for a "steady state" in the conventional way (it doesn't exist), but we must appropriately re-normalize the system, introducing what Barro and Sala-i-Martin (2003) define "control-like" and "state-like" variables. Because both of these variables will be constant along the normalized path of balanced endogenous growth of the economy, we can look for their steady state values, and proceed, to linearize the dynamical system around its (normalized) steady state (as standard in many models of exogenous growth). This linearization enables us to ascertain the nature of the steady state, and therefore to determine the qualitative behavior of the dynamical system at hand, in a neighborhood of its rest point.

To study the system's transitional dynamics, we introduce a control-like, and a state-like variable. Specifically, we define the control-like variable x as the ratio of the consumption to the capital of the representative capitalist. A log-differentiation of x straightforwardly generates its law of motion, that reads

$$x \equiv \frac{c_t^i}{k_t^i} \Rightarrow \frac{\dot{x}}{x} = -\rho + x + y \Rightarrow \dot{x} = (x + y - \rho)x = x^2 + (y - \rho)x. \tag{39}$$

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In addition, we proceed to define the state-like variable y as the ratio of the equilibrium political pressure exercised by capitalist i, and its own capital stock. Again, a log-differentiation of y generates its law of motion, that reads

$$y \equiv \frac{p(k_t^i)}{k_t^i} \Rightarrow \dot{y} = \frac{\dot{k}_t^i}{k_t^i}\left[p'_k(k_t^i) - y\right]. \tag{40}$$

Both equations (39) and (40) will be linearized around their steady state in the Appendix, where the saddle-path's equation, taking the economy to its steady state, will also be computed.

The economy's dynamic evolution is mainly characterized by the next two Propositions.

Proposition 1 *The economy reaches a unique balanced growth condition, for every initial condition, following a saddle-path, and it eventually converges to the unique (normalized) steady state* $\{x^* = \rho, y^* = 0\}$ *. In balanced growth, both the economy's growth rate and the capitalists's welfare are maximized, by taxing at the rate (10). Political pressure remains positive and infinite, but stops growing and therefore becomes negligible as compared to the accumulated stock of capital, whereas the accumulated stock of capital keeps growing at the constant equilibrium rate. This is equal to the consumption growth rate.*

Proof. See Appendix.

Proposition 1 leads to the next Proposition, providing additional characterization of the economy's dynamic behavior in balanced growth.

Proposition 2 *Along the balanced growth path the economy's stock of capital, and the consumption of all individuals (workers and capitalists alike) all grow at the constant rate*

$$\gamma^{\infty} = \frac{\alpha}{1-\alpha}\left[\alpha(1-\alpha)A\right]^{\frac{1}{\alpha}} - \rho > 0. \tag{41}$$

In addition, all capitalists consume a constant fraction ρ *of their wealth, their only source of income, and all workers consume entirely their income, deriving exclusively from labor.*

Proof. Immediate consequence of the previous Proposition.

Notice that the positivity of γ^{∞} is ensured by Condition (1). This Condition guarantees that initial net interest rate (i.e. the one initially applying, in correspondence of $k = k_0$, or $R(k_0)$), is higher than the subjective rate of time preference ρ . Since the net interest rate, applying in balanced growth, is strictly greater than $R(k_0)$, Condition 1 clearly implies that inequality (41), holds $a \;\; fortiori$ [51].

The main result, conveyed by the two just stated Propositions, is thus that the dynamic equilibrium, obtained along the balanced growth path, almost entirely coincides with the equilibrium, obtained in an economy always ruled by a technocratic-oligarchic regime (at no cost). Under this regime the capitalist elite have all political power, captured by lobbying (i.e. the pivotal "voter" has $\sigma^i = 0$), and can therefore implement their preferred fiscal policy. So, for example, on the transitional path the consumption of the representative capitalist (as a share of its wealth) is equal to $c_t^i / k_t^i = \rho - p\left(k_t^i\right) / k_t^i$, and this implies that part of the cost of lobbying is absorbed though a reduction of consumption (and the remaining part through a reduction of investment, of course). This expression eventually leads to the equality $c_t^i / k_t^i = \rho$, obtaining as the balanced growth path is reached, corresponding to the consumption function of any pure rich in Alesina and Rodrik (1994)[52].

3.3 Capital Income Share and Inequality Dynamics

Inspired by Piketty (2014), Milanovic (2014, 2016, 2023), and Bengtsson and Waldenström (2018), we characterize the evolution of the ratio θ_t^k between the (post-tax) income of the capitalist class as a whole and the total (post-tax and public spending) income of workers and capitalists combined (or the post-tax income share of capital). The coefficient θ_t^k is important for various reasons, including the fact, that it may capture relevant aspects of the degree of inequality in the distribution of income[53]. This coefficient formally reads[54]

[51] We remind that the tax rate obtaining, as lobbying goes to infinity (and preferred by the capitalists over any other tax), maximizes the net interest rate. This follows from the assumption stated in (13), and explains the form of the net interest rate reported in (41), as well as why inequality (41) holds, making endogenous growth possible.
[52] This result follows from the equalization of the consumption and wealth growth rate along the balanced growth path, that is ultimately a consequence of Uzawa's theorem. See Acemoglu (2009) on this matter.
[53] As remarked by Saez and Zucman (2020), no unique objective statistic for inequality is available. The GINI coefficient is only one of such measures, and it has its own advantages and disadvantages. It may be therefore useful to pay attention to other potential indicators of inequality.
[54] Notice, that the denominator of this fraction reflects the term $\tau_t k_t$ appearing twice (in absolute value): as a tax on capital income and as "reward" of the factor of production g . Incidentally, we remark that equation (42) corresponds within out setup, to Piketty's First Fundamental Law of Capitalism (i.e. $\alpha = r \times \beta$ in his own notation).

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$$\theta_t^k = \frac{(\alpha A \tau_t^{1-\alpha} - \tau_t) k_t}{\left[(\alpha A \tau_t^{1-\alpha} - \tau_t k_t) + \tau_t k_t + \frac{\lambda^\ell (1-\alpha) A \tau_t^{1-\alpha} k_t}{\lambda^\ell} \right]} = \frac{\alpha A \tau_t^{1-\alpha} - \tau_t}{A \tau_t^{1-\alpha}} = \alpha - \frac{\tau_t^\alpha}{A} \ . \qquad (42)$$

Proposition 3 *The ratio* θ^k *between the (post-tax) income of the representative capitalist and the total (post-tax and public spending) income of the economy, expressed by equation (42), increases during the transitional phase until the balanced growth path is reached, where it becomes constant.*

Proof. See Appendix.

This result highlights the fact that factor shares can change over time, depending on the dynamics of taxes on and off the balanced growth path. Off the balanced growth path, taxes are decreasing because of the increasing political pressure exercised by the capitalists on the government; thereby the (net) interest is increasing, and so are the incentives to save. This is because the increasing political pressure of the capitalists causes their social weight to increase, making fiscal policy become more and more conservative and raising the share of (post-tax) income accruing to capital. This finding is significantly related to some recent literature (e.g. Bengtsson and Waldenström, 2018, in particular), documenting the existence, over the long run, of $robust\ \ co-movements\ \ between\ \ the\ \ capital\ \ income\ \ share\ \ and\ \ income\ \ inequality$ (whether measured by top income shares or by the GINI coefficient). Bengtsson and Waldenström conclude (2018, p. 741) that,

> "with our newly compiled long-run dataset, we have shown that capital shares and income inequality are correlated, even if this relationship varies by region as well as between different time periods. Overall, the results yield support to assertions that the capital-labor split is an important determinant of inequality".

While, as the authors acknowledge, the mechanism explaining this association needs to be further investigated, our paper suggests, that the politics of fiscal policy may be part of the story. In particular, our model suggests the existence of a potential $complementarity$ between factor-income inequality and lobbying. Democratic societies, where capital income is more unevenly distributed (i.e. highly concentrated in the hands of tiny minority, as in our model), are likely to experience greater effort of manipulation of the political process by the capitalist elite. Such elite's effort may well further exacerbate inequality, producing a potentially dangerous loop of capital income concentration, that feeds a political-economic empowerment of the rich at the top of the society. This process, in turn, feeds back into a more unequal capital income distribution, *ad infinitum*.

In addition, Elsby *et al.* (2013), show that, between 1980 and 2013, the capital income share of net income has increased from 35 % to 40%. Interestingly, the period in question coincides with the time of major increment in interpersonal inequality observed in the U.S. While a higher capital income share does not, in principle, necessarily lead to a higher interpersonal inequality, it does so where, like in most modern capitalists societies (as well as in our model), the property of capital is strongly concentrated in the hands of a few rich capitalists. See also Milanovic (2017, Ch. 10, pp. 238-239) on this issue, who argues that the growth of the capital-income share leads to more inequality in the personal distribution of income, if three general conditions are satisfied, all of which apply in our framework. Firstly: capital income must be mainly used to finance investment; secondly, the concentration of capital income must be very high; thirdly, there must be a strong association between capital-rich people and overall income-rich people.

On the balanced growth path, instead, taxes are constant at the specific level preferred by the capitalists. This result reflects, that the political pressure exercised by the capitalists is now constant and equal to zero, in terms of *the capital stock* ; the capital stock, instead, keeps growing forever. Therefore, the capitalists' social weight becomes permanently constant as well, at the maximum possible level (i.e. such that the *de jure* political power of the workers becomes virtually irrelevant). Importantly, the Appendix shows that, as workers become gradually politically less relevant, the income share accruing to labor remains constant over time, positive and equal to $\theta_\infty^\ell = (1-\alpha)$. Instead, as also shown in the Appendix, the income share corresponding to the public good g declines over time, but converges to $\theta_\infty^g = \alpha(1-\alpha)$.

Most importantly, we show in the Appendix that[55]

$$\theta_\infty^k \equiv \lim_{t\to\infty} \theta_t^k = \alpha^2.$$

This result is especially important, since it implies, that capital accumulation does not eventually swallow-up all output. Such outcome may occur in Piketty's basic framework, in

[55] Obviously, we then also have that the sum of the income shares related to the three factors of production used, equals to one in balanced growth, since we have that

$$\theta_\infty^k + \theta_\infty^\ell + \theta_\infty^g = 1.$$

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absence of major exogenous shocks, such as wars; or in absence of a drastic redistribution of income, triggered by a potential "revolution threat" eventually posed by the workers to the capitalists, and curtailing their political power[56].

We specify that in our framework in particular, higher taxes, caused for instance by some exogenous event shocking the economy and empowering the workers, tend to reduce both the interest rate and growth itself, that are jointly endogenous. This mechanism keeps in check the political clout of the capitalists, by reducing capital accumulation and therefore the lobbying activity, so that the rich class cannot become excessively powerful, both economically and politically.

4. Conclusion and directions for future work

We have presented an endogenous growth model, where initial political institutions correspond to a constitutional democracy. Hereby, power's nature and origin is mostly *de jure*, and the majority of the citizenry (i.e. the median voter) is fully in control[57]. However, the balance between the *de jure* and the *de facto* political power changes endogenously, along the equilibrium growth path, as economic development gives to the capitalist elite the incentive to invest more and more in the *de facto* political power. This lobbying activity is implemented in order to prevent both the expropriation of capitalist elite's wealth and the related factor-income redistribution in favor of wages. In the end, the rich minority ends up being fully in control of the polity, and implements a technocratic policy, featuring limited redistribution to the workers. In this context the main holder of the *de jure* power (i.e. the median voter) politically becomes almost irrelevant.

The presented model has a number of limitations and shortcuts, that could be addressed in potential future research. Firstly and foremost, the assumption of one-sided lobbying by the rich elite only could be generalized. I regard this assumption as potentially plausible in certain environments, where for example the working class is disproportionally large, uneducated, and with limited "class consciousness", combined with poor leadership[58]. Elsewhere, the assumption

[56]See Milanovic (2023, Epilogue, pp. 292-293) for a discussion of this issue.
[57] Furthermore, democratic constitutional institutions are "fully consolidated", in the sense that they are not threatened by any form of potential drastic change (e.g. a revolution or a military *coup d'etat*).
[58] All these elements contribute to the explanation of why the working class is unable get organized in a lobby as the capitalists are.

in question may prove less appealing, and a generalization of the model, allowing for two-sided lobbying may therefore appear more reasonable. A somewhat related, but more general issue, is the passive acceptation of the political transformation of *status quo* by the working mass, and the progressive distortion by the rich elite of the original democratic social contract, with the consequent polarization of the functional distribution of income. Such process may potentially trigger at some juncture a revolution of the masses, as in the canonical institutionalist setup of Acemoglu and Robinson (2005). It would be very interesting to explore such potential outcome within our model, in partial conjunction with the potentially important role played by "ideology", broadly defined (and including for example religion). As history reveals *ad abudantiam* (see Gramsci, 1971; Piketty, 2020), ideology is itself a significant source of power, that concurs, together with material power, in shaping the whole development path taken by societies[59]. Obviously, the occurrence of a sociopolitical or cultural revolution along the equilibrium path of our model, would delay, or even prevent the emergence of the plutocratic regime, entirely hegemonized by the small rich elite.

The model adopts the Kaldor-Pasinetti type of assumption: according to this assumption workers never save. Again, while some arguments may be made to justify such assumption in our context and even more generally (see Gomes, 2001, and the references cited therein), its generalization might be desirable in future work, treating workers and capitalists more symmetrically. Indeed, our result, that economic growth leads to a smooth empowerment of a small rich elite, should be taken with some caution, despite being *prima facie* consistent with the important patterns of inequality dynamics and redistribution observed in many democracies in the last few decades (e.g. the raise of "fiscal conservatism" or the "raise of the top 1%"). Hereby, we have treated workers as a purely passive subject, whereas a prospective model's extension might partially generalize this result, allowing for instance for bilateral lobbying and giving to the workers some more political voice. In addition, the progressive empowerment of the capitalists increases the interest rate and boosts both the incentive to save and economic growth. As already mentioned at various stages, I don't regard this result as very general and robust, but rather a possible special case of a potentially much broader set of development trajectories. An excessive empowerment of the capitalists (or of part of them) may be detrimental for growth in different setups. For instance, inequality can excessively empower

[59] In his follow-up book, Piketty (2020, Introduction, p. 7) goes as far as saying that inequality is not based on any natural order, but it is rather ideological and political.

incumbent innovators in a Schumpeterian growth model, thereby slowing down the process of creating destruction (e.g. Aghion and Howitt, 1992). In addition, inequality could possibly hinder, at the political level, the implementation of efficiency-enhancing redistributive policies (e.g. Bénabou, 2000). In potential future work, it would also be appealing to allow for a richer set of dynamic economic decisions by the workers, for instance regarding saving and human capital accumulation.

Finally, it would be interesting to understand how the state's fiscal capacity constraint, potentially binding for workers (assuming they had significant political power), would affect the lobbying, redistribution and growth patterns observed in the model. This constraint could potentially lead both to a reduction of the tax rate demanded by the virtual median voter, and of the overall fiscal revenues collected by the government.

Presumably, if the capitalists faced a lower fiscal redistribution potential threat by the workers[60], their incentive to invest in the *de facto* political power would be lower, allowing the virtual median voter to raise its voice in front of the government. Yet, taxes and (indirect) redistribution through higher wages (for given k), may not increase beyond a certain point, due to the relative state's fiscal weakness. Growth, on the other hand, could potentially *increase* to some non-negligible extent, as a result: capitalists would *at one time*, invest less in wasteful *de facto* power acquisition (see also Barro, 2000), *and* experience lower taxation[61]. If this conjecture were correct, one conclusion would follow: that a weaker state, as compared to a stronger one, might lead to a *higher* economic growth rate. In a strong state realm, the virtual median voter may not be able to commit to demand a lower tax rate than its (relatively high) preferred excise, thereby forcing the rich elite to engage in wasteful influence expenditures. On the other hand, one should bear in mind that, in principle, the state capacity constraint could actually be too stringent. This occurs in a failed or quasi-failed state, located outside the "narrow corridor of liberty" (see Acemoglu and Robinson, 2019), whereby the failure of the state to provide valuable public goods can drastically limit, or even almost inhibit economic growth at all. A tentative conclusion, in this regard, could therefore be, that an intermediate level of fiscal capacity, might be preferable, in some circumstances, to both a very high and a very low state capacity.

[60]Due to the limited ability of the state's fiscal apparatus to tax the citizens (e.g. Acemoglu *et al.*, 2011).
[61]Acemoglu (2010) develops a somewhat related point, illustrating some potential disadvantages for the society of a too much strong state, in terms of overinvestment of resources devoted to the purpose of state capture.

5. Appendix

5.1 Proof of Remark 1

To make progress in the proof, and to simplify the exposition, let us define the following expression, which refers to the inequality (31) reported in the main text, evaluated in the symmetric equilibrium we are considering[62]

$$B\left(\lambda^k p_t^i\right) \equiv \left[\alpha(1-\alpha)A\tau^{-\alpha}\left(\lambda^k p_t^i\right)-1\right] < 0, \tag{44}$$

which highlights that, $in \quad our \quad symmetric \quad equilibrium$, total political pressure at each time is $\lambda^k p_t^i$.

For future reference, we remark here that, around the equilibrium, we have that

$$B'\left(\lambda^k p_t^i\right) = -\alpha^2(1-\alpha)A\tau^{-\alpha-1}\left(\lambda^k p_t^i\right)\tau'\left(\lambda^k p_t^i\right)\lambda^k > 0. \tag{45}$$

The Implicit Function theorem, and in particular the implicit differentiation of equation (31) with respect to k_t^i, around the equilibrium point, imply that

$$B\left(\lambda^k p_t\right)\left[\tau'\left(\lambda^k p_t^i\right)\right]^2 \frac{\partial p_t^i}{\partial k_t^i}\lambda^k k_t^i + \left[-\alpha^2(1-\alpha)A\tau^{-\alpha-1}\left(\lambda^k p_t^i\right)-1\right]\tau''\left(\lambda^k p_t^i\right)\lambda^k \frac{\partial p_t^i}{\partial k_t^i}k_t^i$$

$$+B\left(\lambda^k p_t^i\right)\tau'\left(\lambda^k p_t^i\right)\lambda^k = 0,$$

an expression which implies that political pressure increases with capital, or[63]

$$\frac{\partial p_t^i}{\partial k_t^i} = \frac{-B\left(\lambda^k p_t^i\right)\tau'\left(\lambda^k p_t^i\right)}{\left\{B\left(\lambda_t^k p_t^i\right)\left[\tau'\left(\lambda^k p_t^i\right)\right]^2 + \left[-\alpha^2(1-\alpha)A\tau^{-\alpha-1}\left(\lambda^k p_t^i\right)-1\right]\tau''\left(\lambda^k p_t^i\right)\right\}k_t^i} > 0. \tag{46}$$

It is convenient to re-write expression (46), dividing numerator and denominator by $\tau'\left(\lambda^k p_t^i\right)$, as

[62] The sign of B is obviously negative since $\tau\left(p_t^i\right)$ is greater than τ^k, B is (see below) decreasing in τ, and, finally B is equal to zero when $\tau = \tau^k$.
[63] Notice that both the numerator and the denominator of the following expression are negative. Also, the terms λ^k reported outside the parenthesis all cancel out.

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$$\frac{\partial p_t^i}{\partial k_t^i} = \frac{-B\left(\lambda^k p_t^i\right)}{\left\{B\left(\lambda^k p_t^i\right)\tau'\left(\lambda^k p_t^i\right) + \left[-\alpha^2(1-\alpha)A\tau^{-\alpha-1}\left(\lambda^k p_t^i\right) - 1\right]\tau''\left(\lambda^k p_t^i\right)/\tau'\left(\lambda^k p_t^i\right)\right\}k_t^i}. \quad (47)$$

Using the equation (14) introduced earlier, it is easy to verify that, around the equilibrium we have that

$$\tau'\left(\lambda^k p_t^i\right) = (\tau^k - \tau^\ell)\, exp\left(-\lambda^k p_t^i\right) \lambda^k,$$

and that

$$\tau''\left(\lambda^k p_t^i\right) = -(\tau^k - \tau^\ell)\, exp\left(-\lambda^k p_t^i\right) (\lambda^k)^2.$$

It follows that $\forall \; p_t^i$, we have that $\tau''\left(\lambda^k p_t^i\right)/\tau'\left(\lambda^k p_t^i\right) = -\lambda^k$, and equation (47) assumes the simpler form

$$\frac{\partial p_t^i}{\partial k_t^i} = \frac{-B\left(\lambda^k p_t^i\right)}{\left\{B\left(\lambda^k p_t^i\right)\tau'\left(\lambda^k p_t^i\right) + \left[\alpha^2(1-\alpha)A\tau^{-\alpha-1}\left(\lambda^k p_t^i\right) + 1\right]\lambda^k\right\}k_t^i} > 0. \quad (48)$$

Equation (48) reflects a noteworthy result: the capitalists invest in lobbying in order to alleviate the potential fiscal pressure exercised on them by the virtual median voter. A pressure that is increasing the richer (and the economy as whole) is represented by the accumulable factor of production (i.e. the higher is k_t^i), by basic Meltzer and Richard's (1981) logic. In other words, lobbying increases as the "representative" capitalist becomes richer; therefore, together the growth of its wealth, its willingness to protect it as much as possible from the government also increases.

5.2 Proof of Remark 2

We seek to compute the expression of $\partial p_t^i/\partial \Delta$.

For convenience we report again equation (31), defining implicitly the symmetric equilibrium level of taxation, $\tau\left(\lambda^k p_t^i; \Delta\right)$, as a function of the pressure exercised by capitalist i , and depending also on the exogenous "fiscal exploitation" parameter Δ or

$$\underbrace{\left[\alpha(1-\alpha)A\tau^{-\alpha}\left(\lambda^k p_t^i; \Delta\right) - 1\right]}_{-} \underbrace{\tau'\left(\lambda^k p_t^i; \Delta\right)}_{-} = \frac{1}{k_t^i}. \quad (49)$$

The notation used above highlights the fact τ depends on the variable p_t^i, but also, parametrically, on Δ.

Also for convenience, we recall the equilibrium expression for $\tau(\lambda^k p_t^i;\Delta)$, or

$$\tau(\lambda^k p_t^i;\Delta) = \tau^k + \Delta\, exp(-\lambda^k p_t^i). \tag{50}$$

Additionally, we remark that equation (14) implies that

$$\partial\tau(\lambda^k p_t^i;\Delta)/\partial\Delta = exp(-\lambda^k p_t^i) > 0,\ \partial\tau(\lambda^k p_t^i;\Delta)/\partial p_t^i = -\Delta\, exp(-\lambda^k p_t^i)\,\lambda^k < 0,$$

and that

$$\partial^2\tau(p_t^i;\Delta)/\partial p_t^{i2} = \Delta\, exp(-\lambda^k p_t^i)\,(\lambda^k)^2 > 0,\ \partial^2\tau(p_t^i;\Delta)/\partial p_t^i\partial\Delta = -\,exp(-\lambda^k p_t^i)\,\lambda^k < 0.$$

Differentiating implicitly expression (49) with respect to Δ, we obtain that

$$\underbrace{-\alpha^2(1-\alpha)A\tau^{-\alpha-1}(\lambda^k p_t^i;\Delta)\left[\tau_p^{'}(\lambda^k p_t^i;\Delta)\right]^2}_{(-)}\ \lambda^k\frac{\partial p_t^i}{\partial\Delta}$$

$$+\underbrace{\left[\alpha(1-\alpha)A\tau^{-\alpha}(\lambda^k p_t^i;\Delta)-1\right]}_{(-)}\ \underbrace{\tau_{pp}^{''}(\lambda^k p_t^i;\Delta)}_{(+)}\ \lambda^k\frac{\partial p_t^i}{\partial\Delta}$$

$$=\underbrace{\alpha^2(1-\alpha)A\tau^{-\alpha-1}(\lambda^k p_t^i;\Delta)}_{(+)}\ \underbrace{\tau_\Delta^{'}(\lambda^k p_t^i;\Delta)\tau_p^{'}(\lambda^k p_t^i;\Delta)}_{(-)}$$

$$-\underbrace{\left[\alpha(1-\alpha)A\tau^{-\alpha}(\lambda^k p_t^i;\Delta)-1\right]}_{(-)}\ \underbrace{\tau_{p\Delta}^{''}(\lambda^k p_t^i;\Delta)}_{(+)}\ ,$$

which means that

$$\frac{\partial p_t^i}{\partial\Delta}=\frac{1}{\lambda^k}\frac{\{\alpha^2(1-\alpha)A\tau^{-\alpha-1}(\lambda^k p_t^i;\Delta)\tau_\Delta^{'}(\lambda^k p_t^i;\Delta)\tau_p^{'}(\lambda^k p_t^i;\Delta)+}{-\alpha^2(1-\alpha)A\tau^{-\alpha-1}(\lambda^k p_t^i;\Delta)\left[\tau_p^{'}(\lambda^k p_t^i;\Delta)\right]^2+\left[\alpha(1-\alpha)A\tau^{-\alpha}(\lambda^k p_t^i;\Delta)-1\right]\tau_{pp}^{''}(\lambda^k p_t^i;\Delta)}$$

$$-\left[\alpha(1-\alpha)A\tau^{-\alpha}(\lambda^k p_t^i;\Delta)-1\right]\}\ \tau_{p\Delta}^{''}(\lambda^k p_t^i;\Delta).$$

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The sign of this expression is positive, i.e. $\partial p_t^i / \partial \Delta > 0$, as both the numerator and the denominator of this fraction are negative. We conclude that a higher value of Δ , which as we know reflects a higher potential scope of expropriation of the rich by the poor, induces the the former to invest more in influencing the government.

Next, we can attempt to determine the effect of Δ on the equilibrium difference between the tax rate potentially implemented by the virtual median voter vs. the tax rate emerging from the actual political process of a partially captured democracy. We know that

$$\tau(\lambda^k p_t^i; \Delta) = \tau^k + \Delta\, exp(-\lambda^k p_t^i).$$

Subtracting τ^ℓ from both members, we obtain that

$$\tau(\lambda^k p_t^i; \Delta) - \tau^\ell = \tau^k - \tau^\ell + \Delta\, exp(-\lambda^k p_t^i),$$

or, equivalently,

$$\tau^\ell - \tau(\lambda^k p_t^i; \Delta) = \Delta - \Delta\, exp(-\lambda^k p_t^i),$$

an expression that implies the following result

$$\frac{\partial[\tau^\ell - \tau(\lambda^k p_t^i; \Delta)]}{\partial \Delta} = 1 + \Delta\, exp(-\lambda^k p_t^i)\, \lambda^k \frac{\partial p_t^i}{\partial \Delta} > 0,$$

obviously equivalent to expression (33) reported in the main text.

We conclude that an increment in policy polarization index Δ , $magnifies$ the difference between the virtual median voter's preferred taxes, and the actual taxes implemented by the political system under political pressure, i.e. it increases in some sense the scope and effectiveness of lobbying.

5.3 Proof of Remark 3

For convenience, we again report below equation (31), or

$$[\alpha(1-\alpha)A\tau^{-\alpha}(\lambda^k p_t^i) - 1]\tau'(\lambda^k p_t^i)k_t^i = 1, \tag{52}$$

as well as equation (48),

$$\frac{\partial p_t^i}{\partial k_t^i} = \frac{-B\left(\lambda^k p_t^i\right)}{\left\{B(\lambda^k p_t)\tau'\left(\lambda^k p_t^i\right) + \left[\alpha^2(1-\alpha)A\tau^{-\alpha-1}\left(p_t^i\right)+1\right]\lambda^k\right\}k_t^i} > 0. \tag{53}$$

Because both the numerator of this expression is positive, and so are each term of the denominator, we can divide both the numerator and the denominator by $-B\left(\lambda^k p_t^i\right)$, and re-write the whole fraction as

$$\frac{\partial p_t^i}{\partial k_t^i} = \frac{1}{-\tau'\left(\lambda^k p_t^i\right)k_t^i + \frac{\left[\alpha^2(1-\alpha)A\tau^{-\alpha-1}\left(p_t^i\right)+1\right]\lambda^k}{-B\left(\lambda^k p_t^i\right)}k_t^i}. \tag{54}$$

Moreover, from equation (52), we can write that

$$\frac{1}{-\tau'\left(\lambda^k p_t^i\right)k_t^i} = \left[\alpha(1-\alpha)A\tau^{-\alpha}\left(\lambda^k p_t^i\right) - 1\right].$$

Recall now that, by the assumption concerning the limit behavior of the tax function, we have that

$$\lim_{P_t^k\to\infty} \tau(P_t^K) = \lim_{p_t^i\to\infty} \tau\left(\lambda^k p_t^i\right) = [\alpha(1-\alpha)A]^{\frac{1}{\alpha}} \equiv \tau^k, \tag{55}$$

a result which also implies that

$$\lim_{p_t^i\to\infty}\left[\alpha(1-\alpha)A\tau^{-\alpha}\left(\lambda^k p_t^i\right) - 1\right] = \lim_{p_t^i\to\infty} B\left(\lambda^k p_t^i\right) = 0. \tag{56}$$

But, since

$$\lim_{p_t^i\to\infty} \tau'\left(\lambda^k p_t^i\right) = \lim_{p_t^i\to\infty} \tau'\left(\lambda^k p_t^i\right) = (\tau^k - \tau^\ell)\, exp\left(-\lambda^k p_t^i\right) = 0,$$

it must then be the case that the following limit result applies

$$\lim_{k_i^i\to\infty}\left[-\tau'\left(\lambda^k p_t^i\right)k_t^i\right] = \lim_{k_i^i\to\infty}\left\{-\tau'\left[\lambda^k p\left(k_t^i\right)\right]k_t^i\right\} = \infty,$$

for otherwise equation (52) would fail to hold. Such result, in turn, implies that

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$$\lim_{k_i^i \to \infty} \frac{\partial p_t^i}{\partial k_t^i} = \lim_{k_i^i \to \infty} \frac{1}{-\tau'(\lambda^k p_t^i) k_t^i + \frac{\left[\alpha^2 (1-\alpha) A \tau^{-\alpha-1} (p_t^i) + 1\right] \lambda^k}{-B(p_t^i)} k_t^i} = 0.$$

This is the case since

$$\lim_{k_i^i \to \infty} \frac{\left[\alpha^2 (1-\alpha) A \tau^{-\alpha-1} (\lambda^k p_t^i) + 1\right] \lambda^k}{-B(\lambda^k p_t^i)} = \infty,$$

as the denominator of this fraction tends to zero as k_t^i, and therefore p_t^i (see expression (56) tends to infinity, whereas the numerator clearly tends clearly to a finite number.

This is what had to be demonstrated (see equation (34)).
In addition, using de l'Hospital theorem, we can prove the additional result, that will be useful in the following,

$$\lim_{k_t^i \to \infty} \frac{p_t^i(k_t^i)}{k_t^i} = \lim_{k_i^i \to \infty} \frac{\partial p_t^i}{\partial k_t^i} = 0. \tag{57}$$

5.4 Proof of Remark 4

Recall that the function $R(\cdot)$ is as smooth function of k, defined as

$$R(k) = aA\tau^{1-\alpha}(p(k)) - \tau(p(k)),$$

and it is such that $R(k_0) > \rho$. We have that

$$R'(k) = a(1-\alpha)A\tau^{-\alpha}(p(k))\tau_p'(p(k))p_k'(k) - \tau_p'(p(k))p_k'(k).$$

This expression is equal to

$$R'(k) = \underbrace{\left[a(1-\alpha)A\tau^{-\alpha}(p(k)) - 1\right]}_{(-)} \underbrace{\tau_p'(p(k))}_{(-)} \underbrace{p_k'(k)}_{k} > 0,$$

where the sign of the term in squared brackets is negative since it is equivalent to the term (44), which is negative for any finite k, as we already know.

We conclude that

$$\gamma(k) \equiv aA\tau^{1-\alpha}\big(p(k)\big) - \tau\big(p(k)\big) - \rho > 0,$$

for any $k \in [k_0, \infty]$, as claimed.

5.5 Proof of Proposition 1

We seek to determine the steady state of the normalized system. We begin with the first dynamic equation, and obtain that

$$\dot{x} = 0 \Rightarrow x + y = \rho \Rightarrow x^* = \rho - y^*$$

The second equation instead has the following steady-state, $y^* = 0$, an immediate consequence of the equation (57) above.

We can now linearize the system around its steady-state, $\{x^* = \rho, y^* = 0\}$. The first linearized equation, involving x, reads (see equation (39)),

$$\dot{x} = (2x^* + y^* - \rho)(x - x^*) + x^*(y - y^*) = \rho(x - \rho) + \rho(y - 0). \quad (58)$$

In addition, we remind that the second equation of our dynamical system, involving y, reads, (see equation (40))

$$\dot{y} = \left\{\left[aA\tau^{1-\alpha}\left(p\left(k_t^i\right)\right) - \tau\left(p\left(k_t^i\right)\right)\right] - x - y\right\}\left[p_k^{'}\left(k_t^i\right) - y\right]. \quad (59)$$

We can then linearize this differential equation around its steady state, keeping in mind (see equation (57)) that

$$p_k^{'}\left(k_t^i\right) \to p^\infty = 0,$$

and that (recall the assumption stated in (13))

$$\left[aA\tau^{1-\alpha}\left(p\left(k_t^i\right)\right) - \tau\left(p\left(k_t^i\right)\right)\right] \to R^\infty = \frac{\alpha}{1-\alpha}[\alpha(1-\alpha)A]^{\frac{1}{\alpha}},$$

as $k_t^i \to \infty$, so that

$$\dot{y} = -(R^\infty - \rho - 0)(y - 0) = -(R^\infty - \rho)y. \quad (60)$$

The determinant of the matrix J of the linearized system reads: det $J = -\rho(R^\infty - \rho) < 0$. This

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means that the unique rest point of the normalized dynamical system is $\{x^* = \rho, y^* = 0\}$ and is a saddle-point. For any initial conditions of the system, there exists one and only path leading the economy to the balanced growth state.

The dynamic analysis of the model can be completed by computing the equation of saddle path, i.e. the linear manifold leading to steady state.

To obtain such last equation, we need to solve the system of differential equations just computed, namely the pair of functional equation given by equations (58) and (60). Some simple algebra shows that

$$x_t - \rho = -\frac{\rho}{\gamma^\infty + \rho} y_0 e^{-\gamma^\infty t},$$

and

$$y_t = y_0 e^{-\gamma^\infty t}.$$

Dividing member-by-member the general integrals of the two differential equations from which we departed, we finally obtain the equation of the saddle path, that reads

$$x = -\frac{\rho}{\gamma^\infty + \rho} y + \rho. \tag{61}$$

Equation (61) represents a straight line in the Cartesian space. It is easy to see that its $slope$, measuring the speed of convergence to the statedly state is decreasing in ρ and increasing in γ^∞. Both results are not surprising, as higher rate of temporal impatience clearly makes people prefer current, as opposed to future consumption. Similarly, a higher asymptotic growth rate, γ^∞, will instead speed up convergence, for the opposite reason. The economy moves along this line until the steady state $\{x^* = \rho, y^* = 0\}$ is finally reached.

5.6 Proof of Proposition 3

We remind that the income of the representative capitalist at time $t < \infty$ reads $r^k(\tau_t)k_t^i = [r(\tau_t) - \tau_t]k_t^i$. This expression reflects the factor income of capitalist i owning k_t^i units of capital at time t, when capital income is taxed at the rate τ_t, delivering a net interest rate (the rate of reward of each unit of capital) equal to $r(\tau_t) - \tau_t$. This expression is, as we know, a strictly concave function of τ_t, maximized at $\tau^k \equiv [\alpha(1-\alpha)A]^{1/\alpha}$.

Also remind that the income of the representative worker at time $t < \infty$ reads $r^\ell(\tau_t)k_t = \omega(\tau_t)k_t$. This expression reflects the factor income of worker i owning 1 unit of labor at time t, when labor income (which is a linear function of the aggregate capital stock k_t) is taxed at the rate τ_t, delivering a wage rate (the rate of reward of each unit of labor) equal to $\omega(\tau_t)k_t$. This expression is, as we also already know, strictly increasing in τ_t.

Defining the ratio of the total income of capitalist class vs. the sum of total income of capitalists and workers and of the provision of the public good at time t, as θ_t^k, we have that

$$\theta_t^k = \frac{\lambda^k r^k(\tau_t)k_t^i}{\lambda^k r^k(\tau_t)k_t^i + \tau_t k_t + \lambda^\ell r^\ell(\tau_t)k_t} = \frac{(\alpha A\tau_t^{1-\alpha} - \tau_t)k_t}{\left[(\alpha A\tau_t^{1-\alpha} - \tau_t)k + \tau_t k_t + \frac{\lambda^\ell(1-\alpha)A\tau^{1-\alpha}k_t}{\lambda^\ell}\right]} = \alpha - \frac{\tau_t^\alpha}{A}.$$

In addition, we have that θ_t^ℓ and θ_t^g, that are similarly defined, read, respectively

$$\theta_t^\ell = \frac{\lambda^\ell r^\ell(\tau_t)k_t}{\lambda^k r^k(\tau_t)k_t^i + \tau_t k_t + \lambda^\ell r^\ell(\tau_t)k_t} = \frac{\frac{\lambda^\ell(1-\alpha)A\tau_t^{1-\alpha}k_t}{\lambda^\ell}}{\left[(\alpha A\tau_t^{1-\alpha} - \tau_t)k + \tau_t k_t + \frac{\lambda^\ell(1-\alpha)A\tau^{1-\alpha}k_t}{\lambda^\ell}\right]} = (1-\alpha),$$

$\forall \ t \in R_+$, and

$$\theta_t^g = \frac{\tau_t k_t}{\lambda^k r^k(\tau_t)k_t^i + \tau_t k_t + \lambda^\ell r^\ell(\tau_t)k_t} = \frac{\tau_t k_t}{\left[(\alpha A\tau_t^{1-\alpha} - \tau_t)k + \tau_t k_t + \frac{\lambda^\ell(1-\alpha)A\tau_t^{1-\alpha}k_t}{\lambda^\ell}\right]} = \frac{\tau_t^\alpha}{A}.$$

Taking the time-derivative of the three expressions reported above, we obtain that

$$\dot{\theta}_t^k = -\alpha\frac{\tau_t^{\alpha-1}}{A}\dot{\tau}_t,\ \dot{\theta}_t^\ell = 0,\ \dot{\theta}_t^g = \frac{\tau_t^{\alpha-1}}{A}\dot{\tau}_t. \tag{62}$$

Lastly, we know that along the transitional path (see equation (13)), we have that

$$\dot{\tau}_t = \dot{\tau}(P_t^K) = \tau_{P^K}(P_t^K)\dot{P}_t^K = \tau_{P^K}(P_t^K)\lambda^k\dot{p}_t^i < 0.$$

This result obtains since $\tau_{P^K}(P_t^K) < 0$ (see equation (14)), and since $\dot{p}_t^i = p_k(k_i^i)\dot{k}_t^i > 0$ and $\dot{k}_t^i > 0$, on and off-the balanced growth path. We conclude that the right-hand-side of equation

(62) is positive. Therefore, the post-tax capital income share is increasing along the transitional path to balanced growth (i.e. $\dot{\theta}_t^k > 0$), and the income share corresponding to the productive public good is decreasing (i.e. $\dot{\theta}_t^g < 0$), along the same path.

Furthermore, because

$$\lim_{t\to\infty} \tau_t = \lim_{t\to\infty} \tau\left(P_t^K\right) = \tau^k \equiv [\alpha(1-\alpha)A]^{1/\alpha},$$

since $lim_{t\to\infty} P_t^K = \infty$. We also have that, as reported in the main text,

$$\theta_\infty^k \equiv \lim_{t\to\infty}\left(\alpha - \frac{\tau_t^\alpha}{A}\right) = \alpha^2,$$

a result which implies that the share of total income accruing (post-tax) to capital, remains strictly bounded away from 1 asymptotically, even if it constantly increases over time. In addition, we also have that

$$\theta_\infty^g \equiv \lim_{t\to\infty} \frac{\tau_t^\alpha}{A} = \alpha(1-\alpha).$$

On the balanced growth path instead, we have that $\dot{\tau}_t = 0$, since $\dot{P}_t^K = \lambda^k \dot{p}_t^i = 0$, i.e. the total political pressure exercised by the capitalist class is constant over time. This result obtains since in balanced growth, we have that,

$$\dot{p}_t^i = \lim_{k_i^i\to\infty} \frac{\partial p_t^i}{\partial k_t^i} \dot{k}_t^i = \lim_{k_i^i\to\infty}\left(\frac{\partial p_t^i}{\partial k_t^i}\right)\dot{k}_t^i = 0,$$

which is the case as expression (57) implies that

$$\lim_{k_i^i\to\infty} \frac{\partial p_t^i}{\partial k_t^i} = 0.$$

It follows that in balanced growth taxes are constant, and therefore that income inequality is also constant (i.e. $\dot{\theta}_t^k = 0$). This reflects that once the economy has reached its balanced growth state, the level of taxation permanently corresponds to the technocratic (i.e. growth maximizing), fiscal policy preferred by the capitalists.

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