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AN OPERATIONAL SHADOW-RATE FRAMEWORK FOR POLICY ANALYSIS

ABSTRACT

During the effective lower bound (ELB) era, policy rates became non-informative regarding the monetary stance, motivating the use of shadow rate models. Based on that, this paper aims to offer a comparative assessment of these models into a working taxonomy. We find that estimates converge at monetary policy turning points but diverge in their within-ELB levels, with three-factor specifications yielding systematically shallower troughs than more constrained alternatives. Moreover, the Euro Area displays a deeper and more prolonged negative policy stance than the United States. Overall, shadow rates are best employed as model-contingent summaries within a multi-model framework, with cross-model uncertainty bands reported alongside any point estimate.

Keywords: Effective Lower Bound; Financial Stability; Monetary Policy Stance; Shadow Rates; Term Structure Models
JEL Classification: E43; E52; E58

RIASSUNTO

Un framework operativo di shadow rates per l'analisi di policy

Durante il periodo del cosiddetto effective lower bound (ELB), il contenuto informativo sull'orientamento della politica monetaria dei tassi di interesse ufficiali è significativamente diminuito motivando l'uso di modelli di shadow interest rate. Su questa base, l'articolo offre una valutazione comparativa di tali modelli attraverso una tassonomia operativa. I risultati mostrano che le stime convergono nei punti di svolta della politica monetaria, ma divergono nei livelli all'interno dell'ELB, con le specificazioni a tre fattori che consentono di ottenere valori minimi inferiori rispetto alle alternative più vincolate. Inoltre, l'area dell'euro presenta un orientamento di policy negativo più profondo e più prolungato rispetto agli Stati Uniti. Nel

complesso, i shadow rates si interpretano al meglio come sintesi dipendenti dal modello all'interno di un framework multi-model, con bande di incertezza cross-model riportate in ogni stima puntuale.

1. Introduction

Since 2008, policy rates fell to, or near, zero, making the effective lower bound (ELB) binding. The policy rate does not reflect the true monetary stance when it hits zero, motivating the revival of the "shadow" rate (e.g., Krippner, 2013; Wu and Xia, 2016; Lombardi and Zhu, 2018). Shadow-rate measures are used to gauge stance during ELB periods, bridging conventional and unconventional regimes (Wu and Xia, 2016; Jones, Kulish and Morley, 2021). Different models, though, often deliver divergent estimates, complicating single-series inference (Bauer and Rudebusch, 2016). This measurement problem has direct implications for banking and macroprudential analysis. In ELB regimes, policy also affects banks through expected short-rate paths, term premia, asset valuations, and central-bank balance sheets, with implications for risk-taking and profitability (Borio and Zhu, 2012; Altavilla, Boucinha and Peydró, 2018). Thus, shadow rates provide a compact cross-country stance indicator for the monetary-policy component of banking-sector financial conditions.

Shadow rates also provide a cross-regime monetary-policy metric (Francis, Jackson and Owyang, 2020; De Rezende and Ristiniemi, 2023). This is relevant for interpreting the transition from the prolonged accommodation that followed the global financial crisis (GFC) and the COVID-19 shock to the subsequent phase of policy normalization and tightening (De Rezende and Ristiniemi, 2023; Hohberger, Ratto and Vogel, 2023). More broadly, the post-GFC period of prolonged negative effective policy stances, as measured by shadow rates, provides a way to quantify the extent to which unconventional monetary policy supported activity and the price level relative to counterfactuals without such measures (Mouabbi and Sahuc, 2019; Hohberger *et al.*, 2023). Shadow rates, therefore, remain useful beyond the ELB because they summarize, in short-rate-equivalent terms, information from expected policy paths, term premia, asset purchases, and balance-sheet policies (Mouabbi and Sahuc, 2019; De Rezende and Ristiniemi, 2023). This, in turn, helps characterize the macroeconomic and monetary-policy starting conditions of the recent high-inflation period and the subsequent path of policy normalization and tightening.

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Based on this background, we address how to quantify stance when the ELB binds through a comparative analysis of shadow-rate models. The aim is to provide a taxonomy of shadow-rate approaches and clarify what each one captures. Guided by this taxonomy, the paper offers a cross-country synthesis to deliver policy-relevant insights that single-model or single-country analyses cannot. The operational contribution is to separate robust stance signals from model-specific level estimates before shadow rates are used for monetary, financial-stability, or cross-country banking analysis. We find that (i) shadow-rates align at turning points but diverge in within-ELB levels, (ii) three-factor specifications generally moderate troughs relative to lower-dimensional or tightly constrained variants, and (iii) representative published series suggest that the Euro Area experienced a deeper and more persistent negative stance than the United States.

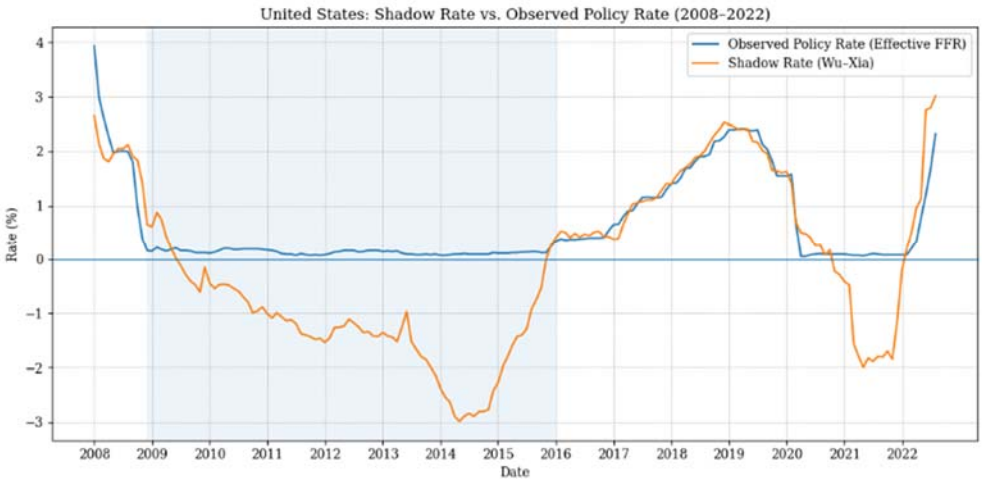
The paper proceeds as follows. Section 2 sets out the ELB intuition. Section 3 organizes the shadow rate models. Section 4 presents the cross-country evidence. Section 5 concludes.

2. ELB AND SHADOW-RATE INTUITION

Interest rates face a floor when agents can hold cash at a non-negative nominal return (Black, 1995). The observed rate is pinned near the floor while the shadow rate moves, recovering within-ELB variation in stance. The shadow short rate, as shown in Figure 1, provides a continuous stance measure across regimes by allowing the latent policy rate to turn negative while the observed rate is censored at the bound (e.g., Black, 1995; Krippner, 2013; Wu and Xia, 2016).

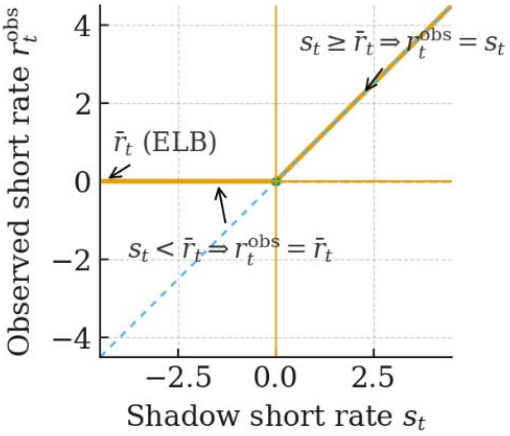
The observed policy rate is $r_t^{obs} = \max\{s_t, \bar{r}_t\}$, where s_t is the shadow rate and $\bar{r}$ the ELB. Above the bound $s_t \geq \bar{r}_t$, r_t^{obs} and s_t coincide; below the bound $s_t < \bar{r}_t$, r_t^{obs} is pinned at $\bar{r}_t$. The truncation is applied inside pricing expectations, embedding the ELB nonlinearity directly into the term-structure model (TSM) (Black, 1995). Figure 2 depicts this mapping.

FIGURE 1 – *United States Effective Federal Funds Rate (FFR) and Wu–Xia Shadow Rate; Shaded Regions Denote ELB Episodes*



Sources: Federal Reserve Economic Data; Wu–Xia Shadow Rate Series.

FIGURE 2 – *Mapping* $\boldsymbol{r_t^{obs} = max\{s_t, \bar{r}_t\}}$: *45° line above* $\bar{r}_t$; *floor at* $\bar{r}_t$ *below*



2.1 Why Standard Gaussian Affine Models Become Problematic at the ELB

Standard Gaussian affine term-structure models (GATSMs) are the workhorse framework for yield-curve analysis in conventional eras (Dai and Singleton, 2000). They combine latent Gaussian factor dynamics with affine no-arbitrage pricing, yielding a tractable representation

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of level, slope, and curvature and a convenient framework for fitting bond yields and extracting expectations and term premia.

In the absence of a lower bound, however, standard GATSMs exhibit specific pathologies in near-zero-rate environments (Christensen and Rudebusch, 2015; Wu and Xia, 2016).

i. Gaussian dynamics allocate non-trivial probability to $s_t < \bar{r}_t$, contradicting the cash-option logic that underpins the ELB.
ii. When r_t^{obs} is pinned at $\bar{r}_t$, volatility collapses beyond the reach of unconstrained models.
iii. ELB episodes feature a flat short end with gradual steepening, which unconstrained affine models fit poorly.

Related evidence also suggests that standard Gaussian-affine yield-based specifications may be too restrictive to capture interest-rate volatility dynamics, especially around stress episodes, because part of volatility risk is not fully spanned by the yield curve and short-rate volatility is not cleanly recoverable from the bond-yield cross section (Collin-Dufresne and Goldstein, 2002; Collin-Dufresne, Goldstein and Jones, 2009; Cieslak and Povala, 2016). By contrast, shadow-rate term-structure models (SRTSMs) generate more accurate short-rate forecasts and more informative estimates of the expected policy path (Bauer and Rudebusch, 2016).

2.2 Shadow-Rate Term-Structure Models

Black's (1995) insight is that fixed-income claims may be priced with reference to a latent short-rate process, while the lower bound is imposed within expectations. This approach embeds the ELB directly in pricing, preserving no-arbitrage while allowing yields to retain information about stance. Although analytical solutions are available in one-factor special cases (Gorovoi and Linetsky, 2004), empirical representations of the yield curve require multi-factor specifications capable of capturing level, slope, and curvature dynamics (Duffie and Kan, 1996; Dai and Singleton, 2000). Other contributions developed tractable approaches to the implementation of SRTSMs, including frameworks that could be extended to richer factor structures:

i. Krippner (2013) develops a bond-option-based non-negative-forward-rate framework that provides a close to Black (1995) framework and is extendable to multi-factor settings.

ii. Wu and Xia (2016) estimate a three-factor state-space SRTSM that embeds the ELB in pricing via a truncated-normal mapping and provides an analytical, highly tractable discrete-time formula for forward rates.

Both classes preserve core properties, namely the elimination of sub-$\bar{r}_t$ outcomes for the observed short rate, endogenous volatility compression at the floor, and an empirically plausible flat short end. A further operational advantage highlighted by macro-finance shadow models is the use of modal (most-likely) policy paths to summarize liftoff expectations when the ELB skews the short-rate distribution. Bauer and Rudebusch (2016) also introduce a tractable "zero lower bound (ZLB) wedge", the gap between fitted and shadow long yields, as a tightness metric for the bound and show that including macro factors materially improves near-ZLB forecasts.

2.3 Model Families and Implementation Choices

Existing models tend to agree on the timing of major turning points while differing more in within-ELB levels (Krippner, 2013; Wu and Xia, 2016; Lemke and Vladu, 2017). Bound-free short-rate-equivalent measures corroborate these dates, providing a useful cross-check that does not rely on an assumed lower bound (De Rezende and Ristiniemi, 2023). Similarly, when examining the divergences within the ELB, models often disagree on the level of accommodation (Bauer and Rudebusch, 2016). These empirical regularities reflect three broad implementation families:

i. Fixed-bound SRTSMs that maintain a constant $\bar{r}_t$ and are commonly estimated with three factors for major jurisdictions, fitting ELB yield curves well.
ii. Time-varying-ELB specifications that allow $\bar{r}_t$ to vary accommodating institutional settings with negative policy rates (Lemke and Vladu, 2017).
iii. Bound-free short-rate-equivalents measures that are constructed without imposing a lower-bound constraint (De Rezende and Ristiniemi, 2023).

3. TAXONOMY OF SHADOW RATE MODELS AND ESTIMATION

The selected literature in this paper is chosen to be representative rather than exhaustive. Contributions that are methodologically influential and widely used in the shadow-rate

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literature are prioritized. Guided by this taxonomy, our evidence map in Section 4 relies on Table 1.

3.1 Pure Term-Structure Shadow-Rate Models

All shadow-rate models inherit no-arbitrage TSM foundations but differ in emphasis. The key distinction is informational. Pure term-structure models recover the shadow rate from the yield curve alone, i.e., they are finance-only models without an explicit macro link, whereas macro-finance versions retain the same ELB logic but use macroeconomic variables or policy-rule structure to help identify the latent policy stance and to support richer policy interpretation. The option-based approach (Black, 1995) imposes the floor within expectations, generating the ELB nonlinearity described by $r_t^{obs} = \max\{s_t, \bar{r}_t\}$. Pricing under truncated short-rate paths yields the characteristic asymmetry at the bound. This lineage extends in two directions:

i. Pure term-structure implementations add factors to fit the cross-section (e.g., Ichiue and Ueno (2007), Kim and Singleton (2012); three-factor AFNS-shadow; three-factor Wu–Xia).

ii. Macro-finance extensions link the latent shadow factor to macro states or rules, enabling policy counterfactuals while preserving the ELB mechanism (Bauer and Rudebusch, 2016; Wu and Zhang, 2019; Jones, Kulish and Morley, 2021)[1]. In both, the shadow rate serves as a single, policy-facing stance index consistent with Section 2's mapping[2].

Beyond TSMs, a statistical, balance-sheet-sensitive shadow policy rate can be extracted from a large monetary data set via a DFM that treats the FFR as missing at the ELB (Lombardi and Zhu, 2018). This bound-free construction preserves pre-ELB tracking of the FFR while mapping QE and balance-sheet operations into a short-rate-equivalent series.

[1] Mouabbi and Sahuc (2019) provide a Euro-Area implementation in a medium-scale DSGE, using a common component extracted from multiple shadow-rate series as the policy instrument to run ELB counterfactuals and quantify macro effects.

[2] A complementary strand dispenses with an explicit lower bound and treats the shadow rate as a market-based stance index inferred from short-rate expectations and yield factors around policy announcements, thus remaining valid outside ELB episodes (De Rezende and Ristiniemi, 2023).

TABLE 1 – *Shadow-Rate Model Families and Key Features*

Model family	Illustrative benchmark papers	ELB spec	Factors	Macro link	Estimation	Typical use	Notes on shadow-rate level
Pure SRTSM (Black/ Gaussian)	Black (1995); Gorovoi and Linetsky (2004)	Fixed	1-2	No	Analytical (1-factor); state-space (EKF) for multi-factor	Intuition/ primer	Levels are highly sensitive to factor count and constraints
Shadow-rate GATSMs	Krippner (2013); Christensen and Rudebusch (2015); Wu and Xia (2016); Priebsch (2023)	Fixed	1–3	No	Analytical approximation + filtering	Real-time monitoring	Moderates extremes vs 1-factor; good cross-section fit; small approximation errors
Macro/ Structural (explicit macro link)	Wu and Zhang (2019); Jones *et al.* (2021)	Fixed	1–3	Yes	Filtering / structural identification	Policy counterfactuals	Clear mapping to guidance/systematic rules; level depends on structure
Macro-finance SRTSM	Bauer and Rudebusch (2016)	Fixed	2–3 + macro	Yes	State-space (modal path; shadow pricing by approximation)	Liftoff timing; ZLB-tightness ("wedge")	Prefer modal over mean near ELB; caution on level non-robustness; macro improves ZLB forecasts
Time-varying ELB	Lemke and Vladu (2017)	Occasionally time-varying (regime-shift)	2–3	No	Nonlinear state-space	Euro Area baseline	Better fit when deposit facility rate < 0
Bound-free short-rate equivalent	De Rezende and Ristiniemi (2023)	Not imposed	N/A	No	Yield-curve-based affine estimation	Any regime; cross-regime check	Sidesteps ELB calibration; yields short-rate-equivalent index
Statistical / Dynamic Factor Model (DFM)	Lombardi and Zhu (2018)	Not imposed	Many (DFM)	Implicit	DFM with missing-data EM	QE-sensitive stance; VAR shocks	Tracks policy rate pre-ELB; captures QE/balance-sheet; useful for VAR shocks; level depends on balance-sheet mix

Notes: AFNS = arbitrage-free Nelson–Siegel; EKF = extended Kalman filter; DSGE = dynamic stochastic general equilibrium; EM = expectation–maximization; QE = quantitative easing; "ELB: Fixed" = constant lower bound; "Time-varying" = lower bound allowed to move; "Not imposed" = bound-free, with no explicit lower-bound constraint. "Factors: Many (DFM)" denotes many observables loading on a small set of latent factors and is not directly comparable to the N-factor count in TSMs. Pure SRTSM entries denote finance-only term-structure models, i.e., models without an explicit macro link. Macro/structural entries denote models with an explicit macro link; these are often linked to New Keynesian or dynamic stochastic general equilibrium frameworks, but may also rely on structural identification without a full DSGE framework. Statistical/DFM and event-identified entries produce short-rate-equivalent stance indices rather than literal shadow short rates.

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A further distinction within pure TSMs concerns factor dimensionality. One-factor models admit analytical pricing but cannot span level, slope, and curvature (Gorovoi and Linetsky, 2004). Three-factor specifications are therefore the empirical workhorse for major jurisdictions, as they fit ELB yield-curve shapes without generating pathological forward-rate curvature and typically moderate troughs relative to one- or two-factor alternatives (Christensen and Rudebusch, 2015; Wu and Xia, 2016).

3.2 Estimation Techniques and Solution Methods

Casting shadow-rate SRTSMs in state-space form enables likelihood-based inference with nonlinear measurement equations. Wu and Xia (2016) achieve tractable estimation of a three-factor model via Kalman filtering with suitable approximations; Kim and Singleton (2012) and Ichiue and Ueno (2007) illustrate two-factor implementations. These methods reproduce key ELB features highlighted in Section 2 (no sub-$\bar{r}_t$ outcomes for r_t^{obs}, volatility compression at the floor, and a flat short end)[3].

Two complementary solution routes have made multi-factor SRTSMs practical:

i. Analytical approximations (e.g., Krippner, 2013) and cumulant-based expansions (Priebsch, 2023)[4] deliver quasi-closed-form yields and real-time filterability with small pricing errors.
ii. An analytical discrete-time approximation combined with (extended) Kalman filtering (e.g., Wu and Xia, 2016) embeds the ELB nonlinearity in the measurement mapping in a way that is highly tractable, with approximation errors of only a few basis points.

In negative-rate jurisdictions, time-varying-ELB state-space models (e.g., Lemke and Vladu, 2017) let $\bar{r}_t$ move with institutional settings; event-identified, bound-free indices (De Rezende and Ristiniemi, 2023) construct a short-rate-equivalent stance measure without imposing a floor, providing a regime-agnostic robustness check. In practice, the cumulant approach also

[3] A complementary route is a dynamic-factor, missing-data EM approach that infers a shadow policy rate from interest rates, monetary aggregates, and balance-sheet quantities—eschewing an explicit ELB and thereby embedding QE directly in the stance metric (Lombardi and Zhu, 2018).
[4] Priebsch (2023) develops a cumulant-based scheme whose accuracy is about half a basis point up to 10-year maturities in a three-factor Gaussian SRTSM, enabling tractable unscented-Kalman-filter estimation; computational complexity does not grow with the number of factors in the pricing step.

accommodates a non-zero empirical lower bound $\bar{r} = r_{min}$ by simple re-parameterization, preserving tractability.

3.3 Implications of Model Choice

Three implications follow directly from the previous sections and motivate Section 4:

i. Multi-factor SRTSMs better match ELB term-structure shapes, yielding more credible shadow-rate dynamics.
ii. Trough levels are model-sensitive – notably to factor dimensionality and ELB treatment – which cautions against over-interpreting single-series levels.
iii. Analytical-approximation + (extended) Kalman filtering (AFNS-shadow; cumulant-based) is ideal for monitoring/communication with small pricing errors; analytical discrete-time SRTSMs (e.g., Wu and Xia, 2016) are suited for research/benchmarking; time-varying-ELB models apply where policy rates can turn negative; bound-free, event-identified indices provide a cross-regime robustness check.

A final comparability point is important before turning to the evidence map. The series compared in Section 4 are drawn from published studies that differ in sample period, data frequency, yield sets, institutional coverage, and estimation windows; we therefore do not interpret cross-study differences as a clean horse race in which methodology alone explains the results. Our objective is narrower and more operational: to identify which qualitative features of stance measurement are robust across model families when applied in practice – especially turning points, direction of movement, persistence within ELB episodes, and broad cross-country rankings – while treating differences in within-ELB levels as informative model uncertainty. The evidence that follows should therefore be read as a structured comparative synthesis of published operational measures, not as a common-sample contest of estimators[5].

[5] Structural stability is itself an open issue. Kim and Priebsch (2024) document predictive- and Lagrange-multiplier-test evidence of structural breaks in the United States Gaussian shadow-rate term-structure models across pre-ELB and ELB/post-ELB samples, cautioning against assuming time-invariant parameters across regimes.

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4. EVIDENCE MAP

This section assembles published shadow-rate series representative of the model families in Section 3 and reads them through a uniform, cross-jurisdiction lens. The emphasis is on comparative dynamics and turning points; level dispersion within the ELB is treated as informative model uncertainty rather than noise. Since these published series are estimated on non-identical data environments, the comparison is intended to recover robust practical signals.

4.1 Cross-Country Trajectories and Policy Signals

For the United States, representative shadow-rate measures indicate that the zero FFR understated the effective degree of accommodation after the ELB became binding in late 2008. Most series turn negative by mid-2009, remain negative during forward guidance and asset-purchase phases, and return toward zero around liftoff in late 2015. This pattern is broadly shared across model classes. Lombardi and Zhu's (2018) measure declines more sharply during the Federal Reserve's first round of large-scale asset purchases than yield-curve-based alternatives, reflecting its sensitivity to mortgage-backed securities holdings and reserve balances. Bauer and Rudebusch (2016) similarly show that macro-finance shadow-rate models imply a later liftoff than forward-mean-based measures when ZLB skewness is pronounced.

For the Euro Area, shadow-rate measures turn negative when the bound binds – on a later timeline and with a deeper, more persistent trough than in the United States. Time-varying ELB specifications (e.g., Lemke and Vladu, 2017) fit the institutional move into negative policy rates (Euro Area ELB ≈ −11 basis points in September 2014), while AFNS-shadow and event-identified, bound-free indices jointly indicate an extended negative stance through the 2010s and renewed declines amid the 2020 interventions (De Rezende and Ristiniemi, 2023). The unifying narrative is a longer and deeper shadow-rate regime than in the United States. Operationally, this means that policy comparisons based on administered rates alone would understate how much longer and how much more persistently accommodative the euro-area stance remained relative to the United States.

Japan features the earliest and longest ELB episode: shadow-rate estimates have been negative for extended stretches since the late 1990s. Trough magnitudes are model-sensitive as low-

dimensional or tightly constrained specifications can deliver very deep negatives, whereas richer (two- and especially three-factor) implementations yield more moderate levels (Ichiue and Ueno, 2007; Kim and Singleton, 2012). All families agree on the qualitative sequence, with model choice chiefly affecting depth but not the timing of turns. Regarding policy, Japan illustrates clearly why persistence and regime chronology are more reliable guides than the exact depth of individual shadow-rate troughs.

For the United Kingdom, shadow-rate series turn negative in 2009, remain below zero through successive asset-purchase and guidance episodes, and firm as liftoff expectations emerge, with renewed declines around the 2020 interventions (De Rezende and Ristiniemi, 2023). Model choice again affects mainly within-ELB depth rather than turning-point timing. Three-factor specifications yield more moderate troughs than lower-dimensional or tightly constrained alternatives, while bound-free, event-identified short-rate equivalents provide a cross-check that does not impose a specific bound. Overall, the United Kingdom lies between the United States and the Euro Area in persistence, illustrating how shadow rates can support cross-country accommodation rankings even when model-specific trough levels differ.

4.2 Convergences, Divergences, and Policy Interpretation

Shadow-rate series align at major turning points but diverge more in within-ELB levels (Wu and Xia, 2016; Bauer and Rudebusch, 2016; Andreasen and Meldrum, 2019). Event-identified, bound-free indices corroborate these turning points without conditioning on an assumed bound (De Rezende and Ristiniemi, 2023). This distinction is central as directional signals are robust for monitoring and communication, whereas level claims remain model-dependent. Two regularities follow:

i. Factor dimensionality matters as three-factor implementations generally yield more moderate troughs than one- or two-factor or tightly constrained alternatives (Christensen and Rudebusch, 2015; Wu and Xia, 2016).
ii. ELB treatment matters because time-varying ELB models (Lemke and Vladu, 2017) reduce mis-measurement in negative-rate jurisdictions, while bound-free, event-identified indices sidestep bound sensitivity but deliver a short-rate-equivalent rather than a literal shadow short rate; balance-sheet-sensitive, factor-based measures

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register earlier and deeper easing when purchases focus outside Treasuries (Lombardi and Zhu, 2018), whereas yield-curve models mute those phases.

4.3 Operational Reading Rules for Policy Analysis

Operational use should therefore rely on triangulation across model families, combining a monitoring-friendly AFNS-shadow estimate with a Wu and Xia (2016)-type benchmark, a time-varying-ELB specification in negative-rate jurisdictions, and an event-identified, bound-free short-rate-equivalent as a cross-regime check (Christensen and Rudebusch, 2015; Lemke and Vladu, 2017; De Rezende and Ristiniemi, 2023).

Finally, contextualization with balance-sheet information improves the interpretation of deep troughs: reading shadow series alongside balance-sheet-sensitive composites and survey/term-premium-based indicators helps disentangle expected-path movements from term-premium compression (Lombardi and Zhu, 2018) – the two channels emphasized in Section 2. For banking and macroprudential users, this implies reading shadow-rate signals jointly with balance-sheet, credit, profitability, and financial-stability indicators rather than as stand-alone measures of monetary or risk conditions.

5. CONCLUSIONS AND BROADER POLICY IMPLICATIONS

We develop a compact operational toolkit for shadow-rate modeling and show, across jurisdictions, that inferences align at policy turning points while levels often diverge within the ELB. The policy takeaway is pragmatic as shadow rates are most useful as disciplined, model-based summaries of unconventional accommodation when used within a robustness frame.

This interpretation is also relevant for the recent high-inflation episode. The evidence reviewed in this paper suggests that shadow rates are useful not only for measuring accommodation at the ELB but also for interpreting the subsequent exit from that regime. By summarizing the depth and persistence of negative effective policy stances, they help connect the post-GFC and pandemic-era accommodation with the later normalization and tightening cycle. This matters because tightening risks reflect both current administered rates and the prior accommodation embedded in broader monetary and financial conditions.

The relevance of this toolkit extends beyond monetary-policy measurement narrowly defined. In ELB regimes, the administered policy rate may remain unchanged while the monetary environment facing banks continues to evolve through expectations, term premia, asset valuations, and balance-sheet policies. Shadow rates convert these channels into a comparable measure of stance for assessing low-rate pressures and normalization-related vulnerabilities in financial-stability and banking-sector analysis. They are therefore best read as inputs into broader macroprudential assessment, not as stand-alone measures of bank risk.

Three implications follow. First, shadow rates should complement administered rates when the ELB binds. Second, model choice should reflect the policy regime, with fixed-bound three-factor specifications suited to zero-rate settings and time-varying-ELB models to materially negative-rate jurisdictions. Third, shadow-rate estimates should be read within a broader monetary-financial indicator set, with cross-model dispersion treated as uncertainty rather than noise.

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