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THE CAGAN'S MODEL, ITS DEVELOPMENTS, AND THE PATINKIN EFFECT

ABSTRACT

This paper reviews the debate on the stability of the inflation rate during high inflation and hyperinflation which started with Cagan (1956) seminal paper. In that paper Cagan arrives at the determination of the optimal inflation rate and the conditions for stability of inflation rates in situations of explosive price growth. The debate on Cagan's model focused mainly on the type of expectations to be taken as a reference, assuming that in real terms the government deficit is unaffected by the rate of inflation. In this paper we review some of the perspectives that open on debate regarding Cagan's model and hyperinflation if we assume that the government budget does, in fact, change in real terms due to inflation. Such changes can arise from the impact of price growth on the real value of both government revenue, the Olivera-Tanzi effect, and government spending, the Patinkin effect. The extent of this impact is key to define the conditions of stability of the inflation rate (whether high or low) and the amount of seigniorage obtained. The paper shows that during German hyperinflation the Patinkin effect was at work, reducing the real value of the government deficit.

KEYWORDS: Cagan; Inflation; Patinkin Effect
JEL CLASSIFICATION: E10; E31; E58; E62

RIASSUNTO

Il modello di Cagan, i suoi sviluppi e l'effetto Patinkin

In questo lavoro si ripercorre il dibattito sulla stabilità del tasso di crescita dei prezzi nelle fasi di inflazione elevata e di iperinflazione che prese avvio dal contributo di Cagan del 1956. In quel contributo Cagan perviene alla determinazione del tasso di inflazione ottimale e alle condizioni di stabilità dei tassi di inflazione in situazioni di crescita dei prezzi esplosiva. Il dibattito sullo schema di Cagan si è focalizzato soprattutto sul tipo di aspettative da prendere a riferimento. Per contro, in essi si è supposto che l'ammontare del disavanzo pubblico in termini reali non risentisse

del tasso di inflazione. In questo contributo si cerca di mostrare le prospettive che si offrono al dibattito sul modello di Cagan e sulle iperinflazioni quando si assume che il disavanzo pubblico in termini reali si modifica a causa dell'inflazione. Tali modifiche possono derivare dai riflessi della crescita dei prezzi sul valore reale sia delle entrate fiscali, l'effetto "Olivera-Tanzi", che della spesa pubblica, l'effetto Patinkin. L'entità di tali riflessi è importante per poter definire le condizioni di stabilità del tasso di inflazione (alto o basso che sia) e l'ammontare stesso del signoraggio. L'articolo mostra che negli anni immediatamente precedenti l'iperinflazione tedesca l'effetto Patinkin ha ridotto il valore reale del disavanzo pubblico tedesco.

1. Introduction

Studies of hyperinflation reached a turning point with Cagan's seminal 1956 contribution in which, inflation, particularly extreme cases, are traced to the government's need to raise resources through seigniorage to cover public deficits. Inflation, in this context, is equated with a tax. Hence the need to define the optimal rate of inflation, i.e. the rate of inflation which maximises a given amount of seigniorage. Cagan, assuming adaptive expectations, comes to the conclusion that this inflation rate is the inverse of the elasticity of money demand to expected inflation.

The amount of seigniorage relative to the inflation rate can be calculated by a Laffer curve: the Inflation Tax Laffer Curve (ITLC), which increases until it reaches a maximum at the optimal inflation rate and then decreases. Given the bell shape of the ITLC, the same amount of seigniorage can be obtained at either a high or a low rate of inflation. However, in his empirical analysis of seven cases of hyperinflation, first and foremost that of Germany in 1923, Cagan finds that the actual rate of inflation is always higher than both the optimal rate and the stable rate calculated in his theoretical model.

Sargent (1982) later addressed this inconsistency by replacing the adaptive expectations hypothesis with the rational expectations hypothesis. Sargent's conclusion, that, assuming equal seigniorage, the high inflation rate is stable rather than the low inflation rate as in Cagan, clashes, however, with empirical evidence showing that in countries affected by hyperinflation there are phases of high, but not explosive, inflation alternating with phases of hyperinflation.

Attempts to resolve this inconsistency have been made by hypothesizing that expectations are formed through a learning process[1].

The fact remains that Cagan's model and its developments assume that the real public deficit to be covered by seigniorage is constant, i.e. not affected by inflation. In reality, the real value of a country's budget deficit is subject to certain effects of inflation. Firstly the Olivera-Tanzi effect, this results from the delay with which taxes are paid compared to the time when the taxable income is assessed. In the presence of inflation, this delay reduces the real value of tax revenues. If it were the only effect in operation then it would be observed that in reality, in phases of high inflation, the public deficit in real terms widens and, therefore, the government has to resort to higher seigniorage to cover the public finance imbalance.

However, in a 1963 paper on Israeli inflation, Patinkin showed that rising prices affect not only government revenue but also government expenditure in real terms. If the latter effect, known as the Patinkin effect, is greater than the Olivera-Tanzi effect, then in phases of high inflation the government deficit falls and paradoxically there is less need for seigniorage.
When, as historical experience confirms, one takes into account that inflation acts on the public deficit in real terms and that the Patinkin effect may prevail over the Olivera-Tanzi effect, two implications emerge. Firstly, that the optimal inflation rate is inevitably different from the one estimated by Cagan on the basis of the hypothesis of a constant real deficit. In fact it changes in relation to the effects of inflation on the budget deficit. The second is that the conclusions and the reasons that can be given for the stability of a low and a high rate of inflation allowing the financing of the fiscal deficit also change.

In this paper the focus is on this latter aspect. The paper is organized as follows. Section 2 is devoted to the Cagan and Sargent models. Section 3 illustrates the Patinkin effect, while Section 4 provides evidence that the Patinkin effect affected the real deficit of the Weimar Republic, and indirectly the amount of seigniorage needed to cover it.

[1] See Marcet and Nicolini (2003).

2. Recent versions of the monetary hypothesis

Economic studies of hyperinflation were numerous in the years immediately following the German hyperinflation, which was traced by some, such as Bresciani-Turroni (1937), to excess money creation due to the Reich's large deficits, and by others, such as Hellferich (1927), to the depreciation of the mark due to the imposition of Reparations on Germany[2].

After a hiatus of several years, explanations of the causes of hyperinflation were given new impetus in 1956 by an essay by Cagan[3]. In his paper Cagan, analysing the hyperinflation[4] of seven countries[5] (including that of the Weimar Republic) developed a monetary framework in which the role of expectations was taken into account. Cagan, through his analysis, came to the conclusion that in all the cases he considered the ultimate cause of hyperinflation was the creation of money to cover public deficits. Such money creation, in fact, allowed governments to enjoy seigniorage revenues, i.e. the real revenues of the state from the issue of money.

In Cagan (1956)'s model the demand for money in real terms depends only on expected inflation. It is assumed, therefore, that in hyperinflation the other variables that normally affect the demand for money are irrelevant. An increase in expectations of inflation reduces the demand for real money:

$$log\ (m) = -\alpha\pi^e \tag{1}$$

where m denotes money in real terms, $m = \frac{M}{P}$, π^e expected inflation under the assumption of adaptive expectations – i.e. expected inflation is revised by the difference between the current level of inflation and the expected level of inflation – and α the semi-elasticity[6] of money demand with respect to the latter variable.

[2] On Keynes' thought about Reparations see Pittaluga (2022).
[3] See Cagan (1956).
[4] Defined by him as a situation starting in the month that the monthly inflation rate exceeds 50 per cent and ending when the monthly inflation drops below 50 per cent and stays that way for at least a year.
[5] The seven cases of hyperinflation considered by Cagan (1956) are those of Austria (October 1921-August 1922), Russia (December 1921-January 1924), Germany (August 1922-November 1923), Poland (January 1923-January 1924), Hungary I (March 1923-February 1924), Greece (November 1943-November 1944), and Hungary II (August 1945-July 1946).
[6] That is, the percentage change in money in real terms as expected inflation changes by one point.

Using equation (1) we can determine the rate of inflation which allows the government to maximise seigniorage. Seigniorage can be represented as follows:

$$S = \frac{\dot{M}}{P} = \dot{m} + m\pi. \tag{2}$$

where S is seigniorage, $\dot{M}$ indicates the change in the money supply, P the price level, $\dot{m}$ the change in the money supply in real terms, and π the rate of inflation. As equation (2) shows, seigniorage has two components: $\dot{m}$, namely pure seigniorage, and the inflation rate, $m\pi$. In equilibrium, pure seigniorage is zero ($\dot{m} = 0$) and expected inflation is equal to actual inflation.

The optimal inflation rate, i.e., the one that maximises seigniorage, is determined from equation (1). The latter can be rewritten as $m = e^{-\alpha\pi}$ from which it follows that the inflation rate, $m\pi$, is equal to:

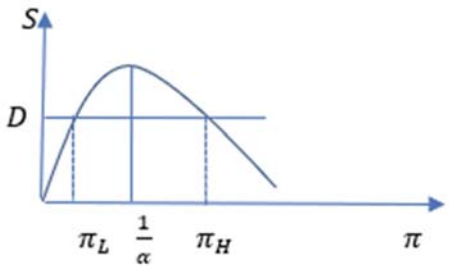
$$S = \pi e^{-\alpha\pi} \tag{3}$$

The level of inflation that maximises seigniorage is obtained by deriving equation (3) with respect to the level of inflation and is equal to the inverse of the semi-elasticity of money demand to inflation:

$$\pi = \frac{1}{\alpha} \tag{4}$$

Figure 1 shows the evolution of seigniorage as a function of the inflation rate. At the level of inflation $\pi = \frac{1}{\alpha}$ is the maximum level of seigniorage. As the figure shows, seigniorage has a bell-shaped form. It increases with an increase in inflation – on the so-called efficient side – until money demand becomes inelastic. When the inflation rate becomes high, the economy begins to operate on the falling "inefficient" side. This means that the same level of seigniorage can be obtained from two different levels of inflation, one low and one high. Seigniorage therefore gives rise to an Inflation Tax Laffer Curve:

FIGURE 1 - *Seigniorage*



Through econometric estimation, Cagan calculated the optimal inflation rate for each of the cases of hyperinflation he considered. In each of them the estimated optimal inflation rate was lower than the actual average inflation rate.

In Cagan's analysis, inflation is due to the financing of a public deficit with money. In which case, seigniorage ($\frac{\dot{M}}{P}$) must equal the government deficit in real terms, D assumed to be constant:

$$\frac{\dot{M}}{P} = D \tag{5}$$

From equation (5) we obtain $\frac{\dot{M}}{M} = \frac{D}{m}$, namely, that the growth rate of money supply depends on the fiscal deficit and the amount of money in circulation in real terms. Using equation (5), the derivative with respect to time of money in real terms, m, is equal to:

$$\dot{m} = \frac{\partial \frac{M}{P}}{\partial t} = D - m\pi \tag{6}$$

Let us first consider the case where expectations are adaptive, the Cagan case, the so-called *constant seigniorage adaptive expectation model.* In this case, expected inflation is revised by the difference between the current level of inflation and the expected level:

$$\dot{\pi}^e = \beta(\pi - \pi^e) \tag{7}$$

where $\dot{\pi}^e$ is the change in inflation expectations while the parameter β determines the speed of adjustment of inflation expectations to the current inflation rate.

We want to obtain the steady-state equilibrium level – that is, the state in which all variables are constant – of the money supply in real terms and the inflation rate. For this, deriving both sides of the money demand equation, equation (1), with respect to time and, using equation (6) and (7), we obtain:

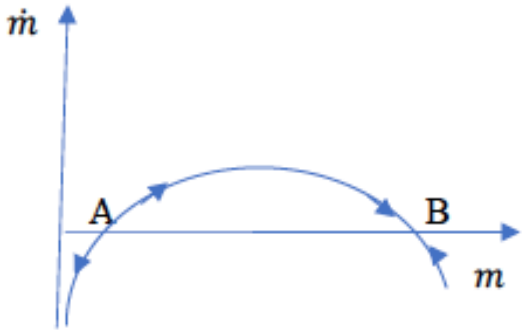
$$m\pi = \frac{1}{1-\alpha\beta}\left(D + \beta m log(m)\right) \tag{8}$$

Substituting equation (8) into equation (7) we have:

$$\dot{m} = \frac{-\beta}{1-\alpha\beta}\left[\alpha D + m log(m)\right] \tag{9}$$

Figure 2 shows the phase diagram corresponding to equation (9) in the case in which $1 - \alpha\beta > 0$[7].

FIGURE 2 - *Phase Diagram of the Constant Seigniorage Adaptive Expectation Model*



In the steady state, which is when all variables are constant, $\dot{m} = 0$. As Figure 2 shows, there are two possible steady states, points A and B. In steady state B, which is characterized by a high quantity of money in real terms, the inflation rate is low. This is because, using equation (8) and the fact of being in the steady state, i.e. $m\pi = D$, given that the level of deficit to be financed, D, is constant, a high level of m corresponds to a low level of inflation. The steady state characterized by low inflation, as shown by the arrows in Figure 2, is stable: if, for some reason, money differs

[7] The case $1 - \alpha\beta < 0$ is not analysed as it does not involve hyperinflation. In this case the graph faces upwards: you have a stable equilibrium characterized by high inflation, and an unstable one with low inflation. You may have a deflationary bubble.

from the equilibrium, market forces will tend to bring it back towards the equilibrium level. The steady state B corresponds, in Figure 1, to the level of inflation π_L.
Equilibrium A, characterized by high inflation, is unstable. If the economy finds itself with a level of money in real terms lower than that corresponding to point A, then as there is high inflation despite a constant fiscal deficit the economy is in an inflationary bubble. Steady state A corresponds, in Figure 1, to the level of inflation π_H.

It can be concluded, therefore, that if expectations are adaptive and if $1 - \alpha\beta > 0$ (i.e. if expectations slowly adjust to the actual rate of inflation and the sensitivity of the demand for money to inflation is low), of the two levels of inflation that ensure the financing of a given amount of fiscal deficit, the stable level of inflation is the lower one. This corresponds, in Figure 1, to the level of inflation on the rising side of the Laffer curve, the level of inflation π_L. It has already been mentioned that, according to Cagan's estimates, the level of inflation in Germany was higher than the level of inflation corresponding to the maximum level of seigniorage. This led Cagan to conclude that Germany, like the other six countries in his study, at some point

"[i]n order to compensate for the low level to which the tax base fell after many months of hyperinflation, the tax rates rose to astronomical heights"[8],

i.e. was on the wrong side of the Inflation Tax Laffer Curve. This is Cagan's paradox.

An explanation of why Germany was on this side of the Laffer curve emerges using the model proposed by Sargent and Wallace (1987). They replace the adaptive expectations used by Cagan with rational expectations. By doing so, Sargent and Wallace (1987) reach very different results from Cagan's. Incorporating the rational expectations hypothesis they assume that people make predictions consistent with economic theory using all available information. In this setting people make forecast errors but these errors are not predictable given the information available. In a setting where variables are not stochastic, the assumption of rational expectations coincides with assuming perfect forecasts. Equation (7) is replaced with:

$$\pi^e = \pi \tag{10}$$

[8] See Cagan (1956; p. 89).

The model is solved similarly to the case of adaptive expectations. Substituting equation (10) – i.e. rational expectations – into demand for money, equation (1), and multiplying both sides of the latter by m, one obtains:

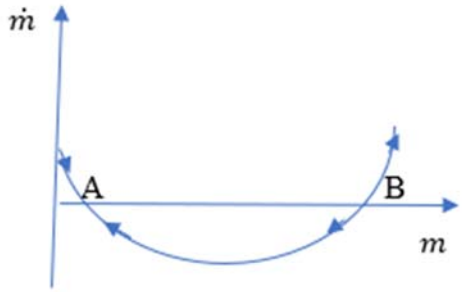
$$m\pi = -\frac{1}{\alpha} mlog(m) \tag{11}$$

Substituting equation (11) into (7), we obtain:

$$\dot{m} = f + \frac{1}{\alpha} mlog(m) \tag{12}$$

Figure 3 shows the phase diagram of equation (12). As the Figure shows, there are two equilibria in this case as well. Equilibrium A, characterized by high inflation, is stable, while equilibrium B, characterized by low inflation is unstable. In this model there cannot be a hyper-inflationary bubble, but, there can be a hyper-deflationary bubble. In the case of rational expectations[9], therefore, the stable inflation level is the high inflation level, corresponding, in Figure 1, to the inflation level π_H, on the downward sloping side of the Laffer curve. Thus, according to Sargent and Wallace's model, it is quite natural for inflation to rise above the level that maximises seigniorage.

FIGURE 3 - Phase Diagram of the *Constant Seigniorage Rational Expectation Model*



The explanation of cases of high inflation with the existence of an Inflation Tax Laffer Curve characterized by a stable equilibrium on the wrong side of the curve, i.e. characterized by high inflation, has been questioned in several articles both from theoretical and empirical view

[9] Or of adaptive expectations and $1 - \alpha\beta < 0$.

points[10]. At the theoretical level, for example, Gutierrez and Vazquez (2004), starting from the two main models used to explain the demand for money – i.e. the cash-in-advance model and the money in the utility function model – show under which conditions the Inflation Tax Laffer Curve can arise. They show that in the cash-in-advance model the Inflation Tax Laffer Curve arises only if the relative risk aversion parameter of the utility function of the representative households is less than one. This means that it can arise only for unrealistically high intertemporal substitution for consumption. On the contrary, for reasonable values of the relative risk aversion parameter, their model leads to a unique unstable steady state, characterized, therefore, by explosive inflation. A similar result occurs with the money in the utility function model. In this case they conclude that explosive hyperinflation dynamics are more likely when the transaction role of money becomes important.

While Gutierrez and Vazquez (2004) show how explosive inflation is likely, Marcet and Nicolini (2003) explain the existence of recurrent episodes of hyperinflation by changing the type of expectations. In particular, they replace the assumption of rational expectations with the assumption of boundedly rational learning rules: expectations are not fully rational, but only small deviations from rationality in adaptive learning are allowed. They also assume that the government finances the budget deficit by seigniorage only when inflation is below some certain limit and establishes a fixed exchange regime otherwise in order to stabilize hyperinflation. The model implies that the low inflation equilibrium is locally stable but various shocks may push the inflation rate out of the range that allows the restricted adaptive learning mechanism to bring the economy to the low inflation equilibrium. The problem with this model is that it relies on the assumption that the government can temporarily stabilize hyperinflation by setting the exchange rate peg[11].

Both Gutierrez and Vazquez (2004) and Marcet and Nicolini (2003) maintain the assumption that the government deficit to finance does not depend on the level of inflation. In the next section we show what happens when this assumption proves false.

[10] While Edwards and Tabellini (1991), Bali and Thurston (2000), Fisher *et al.* (2002) and, more recently, various papers on east European countries (see, for example, Petrović and Mladenović, 2022) have found confirmation of the Inflation Tax Laffer Curve, others, such as Eckstein and Leiderman (1992) and Bental and Eckstein (1997) find that seigniorage remained trendless in the case of Israeli inflation.

[11] Marcet and Nicolini (2003) explain recurrent inflation by introducing boundedly rational learning rules. Zarazaga (1995) reached a similar result – i.e. the presence of recurring episodes of hyperinflation and chronic high inflation – with a game theory approach where several local policymakers with conflicting interests compete for seigniorage.

3. THE OLIVERA-TANZI AND PATINKIN EFFECTS

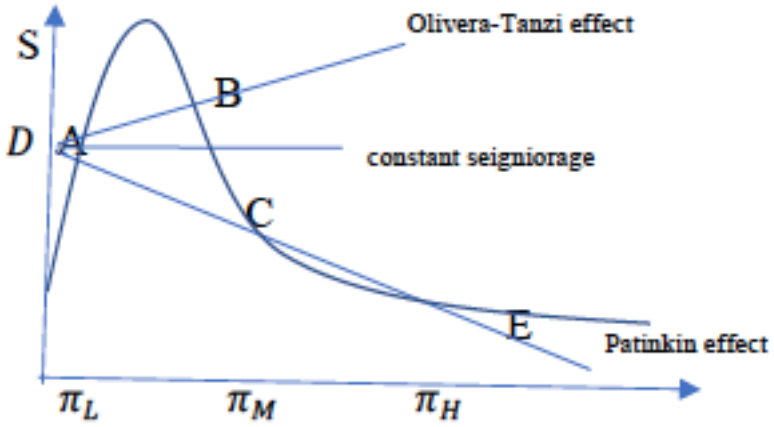
So far, we have assumed that the level of government deficit to be financed was, in real terms – the variable D in the previous model – constant; in particular, it did not depend on the level of inflation. However, the literature has shown at least two mechanisms through which inflation impacts on the real budget deficit. The most frequently cited is the Olivera-Tanzi effect (Olivera, 1967; Tanzi, 1978) according to which real budget revenues decline as inflation rises because of delays in tax collection. This effect predicts that real tax revenues decline as inflation rises and thus the budget deficit is higher at higher inflation rates. This effect destabilizes public finance and, therefore, strengthens the incentives to stabilize inflation.

While the Olivera-Tanzi effect has been widely studied and has been the subject of numerous empirical analyses, the other effect, the Patinkin effect, has received less attention. According to this effect – first highlighted by Patinkin (1993) and then developed by Cardoso (1998) – inflation reduces real public expenditure. Why can this happen? First, real interest rates decline with inflation, reducing the "cost" of servicing debt. Second, during periods of high inflation local governments may delay wage payments. These delays reduce real expenditure. Lastly, governments program expenditure using a forecast for inflation that is usually lower than actual inflation. This leads to actual expenditure being lower than programmed expenditure.
Due to the Patinkin effect the budget deficit declines with inflation. Thus, the Patinkin effect acts as a stabilizer on fiscal imbalances and weakens the incentives of policymakers to fight inflation.

According to Cardoso (1998), the Patinkin effect prevails over the Olivera-Tanzi effect at high inflation rates. If this happens, real expenditure is lower than it would be without inflation and real expenditure increases when inflation falls. As a result, the fiscal adjustment needed once inflation falls is usually higher than expected.

If we introduce the Olivera-Tanzi effect and the Patinkin effect to the model from the previous section how does it change? According to the Olivera-Tanzi effect the budget deficit increases with inflation. In this case the budget deficit curve will have a positive slope (Figure 4). The Olivera-Tanzi effect does not change the basic result of the previous model, i.e. the presence of two steady states (A and B). As before, the stability of the two steady states depends on expectations.

FIGURE 4 - *Seigniorage, Olivera-Tanzi Effect and Patinkin Effect*



The case of the Patinkin effect is different, in fact, there are three possible steady states (A, C, E). This is due to the fact that seigniorage, after a certain amount of inflation, decreases slowly with inflation (after a certain level of inflation the seigniorage curve becomes convex). In the case of adaptive expectations, for instance, the steady states with low or extremely high inflation are stable, while the equilibrium that corresponds to the middle inflation is unstable. With adaptive expectations the model rules out explosive inflation: no matter how high the budget deficit is, there is always at least one stable equilibrium with extremely high inflation[12].

Cardoso (1998) considers the two effects together, the Olivera-Tanzi effect and the Patinkin effect. She first defines "virtual deficit" as the deficit that would be observed at zero inflation. She assumes that when inflation is positive, the budget deficit depends on inflation and, according to her, the relationship between budget deficit and inflation is not linear. At low inflation rates the budget deficit increases with inflation since the Olivera-Tanzi effect predominates over the Patinkin effect. At high inflation rates, the Patinkin effect predominates over the Olivera-Tanzi effect and the budget deficit decreases with inflation. At even higher inflation rates, the Olivera-Tanzi effect predominates again and the budget deficit again increases with inflation. In short, the response of the budget deficit to inflation depends on how strong the Patinkin effect is relative to the Olivera-Tanzi effect at different levels of inflation.

[12] In the case of rational expectation, instead, the stable equilibrium is the one in the middle.

She represents the link between budget deficit and inflation with a cubic function:

$$D(\pi) = D_0 + a\pi^3 - b\pi^2 + c\pi \tag{13}$$

where D_0 is the virtual deficit. Assuming adaptive expectations, Cardoso finds three steady state equilibria. A stable equilibrium characterized by low inflation, an unstable one in the middle, and another stable equilibrium for a high inflation rate. The last of these corresponds to a situation where inflation is high, seigniorage is low but sufficient to finance the real public deficit since it, in turn, has been eroded by inflation.

From these theoretical conclusions some policy indications can be drawn, in particular, during an inflation stabilization attention should be paid to the virtual government deficit. In fact, when the Patinkin effect is strong, fiscal adjustment may have to be more marked than projected because the budget deficit may increase with declining inflation.

The importance of the Patinkin effect is also addressed in a paper by Pekarski (2011). While Cardoso considers the whole budget deficit to be subject to the Patinkin effect, Pekarski argues that only a part of the deficit may be affected by inflation while the rest of the budget deficit is not. An example of "inflation-proof" deficit is, for example, debt serviced in a foreign currency or indexed debt. Pekarski is interested in the impact of fiscal policy on inflation, disentangling the effects of changes in exposed and inflation-proof parts of the total deficit. Changes in different items of the budget balance sheet may have very different effect on inflation[13], therefore, looking for inflationary effects of fiscal policy using aggregate budget measure may be misleading. An important implication for the design of stabilization policy is that inflation-exposed expenditure should be cut first. According to Pekarski, the budget deficit depends on inflation in the following way:

$$D(\pi) = d_E(1 - \beta\pi) + d_P \tag{14}$$

where $D_0 = d_E + d_P$ is Cardoso's virtual deficit, i.e. the deficit that would have occured with zero inflation. As just noted, Cardoso considers the budget deficit as subject to the Patinkin effect. Pekarski, instead, assumes that only a certain part of this zero-inflation deficit, i.e. d_E, is affected

[13] On this aspect see also Heymann and Sanguinetti (1994).

by inflation. The other part, d_P represents the inflation-proof deficit. The parameter β represents the strength of the Patinkin effect.

Pekarski's main finding is that while the relationship between the exposed deficit and a stable inflation rate is ambiguous, the relationship between the inflation-proof deficit and a stable inflation rate is always positive. Therefore, even small increases in the inflation-proof part of the budget deficit may trigger extremely high inflation in a country characterized by moderately high inflation. At the same time, a decrease in the inflation-proof part may act as a stabilizer of hyperinflation, even if the changes are minor[14].

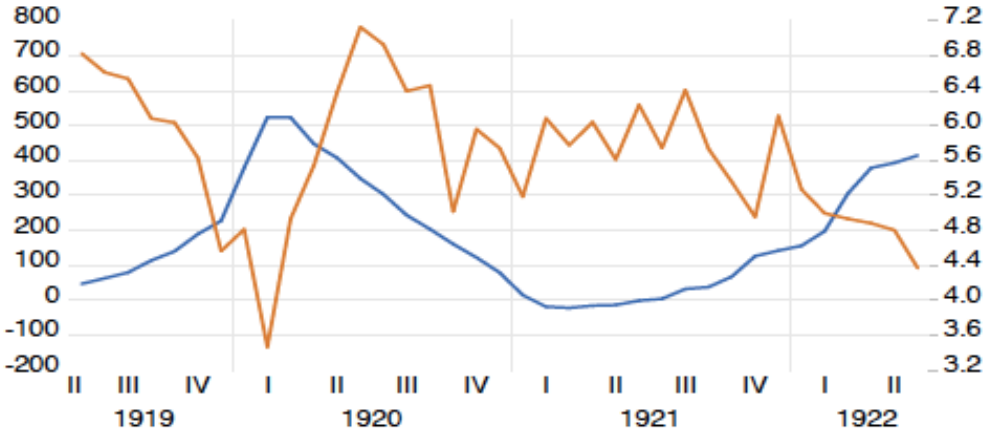
4. The case of Weimar German inflation

As Figure 5 shows, between 1919 and June 1922 – i.e. before the onset of hyperinflation – Germany's year-on-year inflation rate was very high, often exceeding 100 percent. Two phases of intense acceleration of price growth can be identified: one in the second part of 1919 and the other in the second part of 1921. Both of these phases of accelerating price growth were preceded by a pronounced depreciation of the mark. The first phase of depreciation of the German currency took place in July 1919 immediately after the signing of the Treaty of Versailles when the exchange rate for the Mark went from 3.34 to 11.14 per dollar between June and December.

The second phase of pronounced depreciation of the German currency occurred in the middle of 1921, after the definition of the London Schedule of Payments (May 5, 1921). This phase intensified further in the autumn, when the Allies decided to transfer Upper Silesia, an industrial region rich in coal mines, from Germany to Poland. Between the end of May and the end of November, the mark's exchange rate against the dollar rose from 242 to 1175. This significant depreciation of the mark led to an increase in import prices (by 271.8 percent), followed by an increase in domestic prices (by 161.1 percent) and in the cost of living (by 65.8 percent).

[14] Pekarski's policy implications can be useful for inflation episodes in developing countries. For the case of Zimbabwe see Saungweme and Odhiambo (2020, 2021).

FIGURE 5 - *German Public Deficit and Inflation*



Source: Bresciani-Turroni (1937).
Legend: The blue line is annual inflation (measured on the left scale) while the red line is the logarithm of the public budget deficit (measured on the right scale).

In the period between the two phases of pronounced depreciation, i.e. between March 1920 and June 1921, the mark appreciated by 7.5 percent against the dollar. There were several reasons for this[15], on one hand, Germany, as an importer of commodities and exporter of manufactured goods, benefited from an improvement in the terms of trade due to the fact that relative commodity prices fell more sharply than manufactured goods prices at that time. While, on the other, confidence in the mark rose as the failure of the "Kapp" coup d'état gave many people the impression that Germany had acquired political stability. This resulted in large inflows of foreign capital[16].

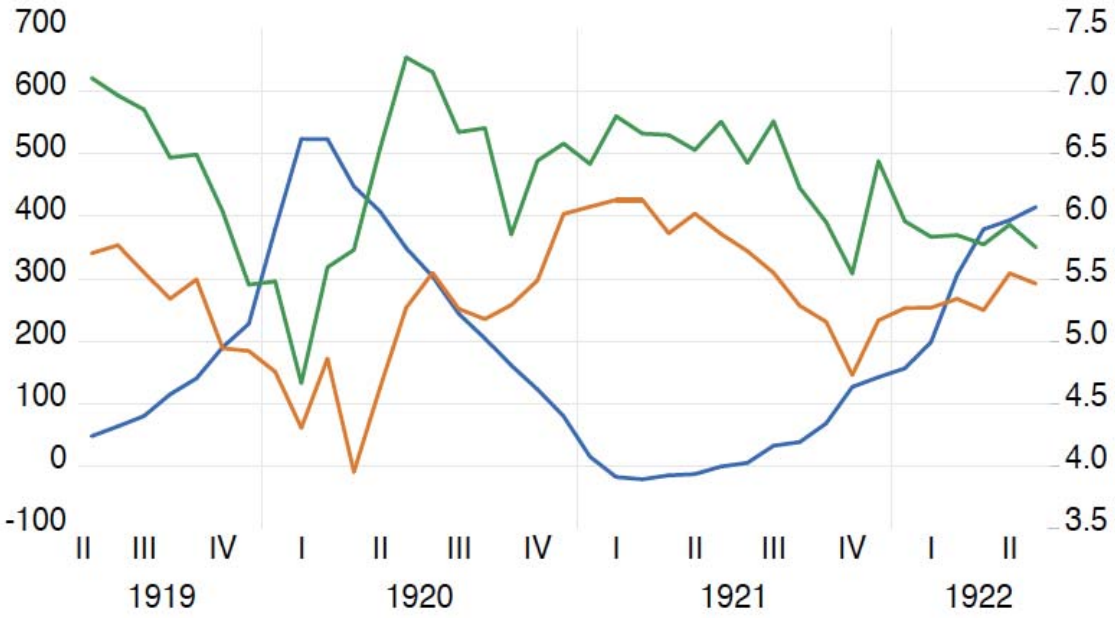
The appreciation of the mark, the post-war slump abroad and the world-wide fall in the prices of food and raw materials favoured between March 1920 and June 1921 a significant fall in prices in Germany: in this period, in fact, wholesale prices in this country fell by 20.0 per cent and retail prices by 22.7 per cent and the German economy enjoyed relative stability during this period[17].

[15] See Graham (1930).
[16] "*As business conditions/opportunities deteriorated in the great postwar depression of 1920/22, however, foreign investors, particularly in the United States and Spain, looked to Germany as a place where they might employ their money profitably*". See Feldman (1982; p. 185).
[17] Several scholars have viewed this period of "relative stabilization" as a lost opportunity to stabilize the German economy (Bresciani-Turroni, 1937; Nurkse, 1946 and Ferguson, 1997). The German government made no attempt in this direction. It simply remained passive: the political conditions for a stabilization of the German economy were

The different phases of inflation just illustrated were reflected in the trend of the German government deficit in gold marks (which can be considered a good proxy for the real value of this variable). Figure 5 shows that the budget deficit in gold marks declined during the phases of price increases, particularly in the second half of 1919 and between the second part of 1921 and the beginning of 1922.

Figure 6 shows that the trend of the government deficit just considered depended, to a large extent, on the trend of public expenditure in gold marks. The latter tending to contract more pronouncedly than government revenue in gold marks in times of rising prices. Therefore, it can be said that in the years before the hyperinflation of 1923, in Germany, the Patinkin effect was greater than the Olivera-Tanzi effect.

FIGURE 6 - *German Inflation, and Government Revenue and Expenditure*



Source: Bresciani-Turroni (1937).
Legend: The blue line is annual inflation (measured on the left scale); the red line is government revenue while the green line is government expenditure (both measured on the right scale).

lacking. Between the spring of 1920 and the early summer of 1921 no stabilization program was initiated or even discussed. The justification given by the political class for this passive behavior was that the Reparations problem had to be solved before stabilization measures could be taken (Feldman, 1982; p. 190). In reality, the Republic was not strong enough to withstand the resistance, protests and unrest that would have occurred in parts of German society following spending cuts and tax increases.

This was to a large extent due, as Table 1 shows, to the composition of the Reich's expenditure, which changed markedly after WWI in favour of current expenditure and transfers at the expense of capital expenditure. The latter, between 1913, 1925 and 1932, fell from 19 to 6 percent and finally reached a mere 3 percent of the total.

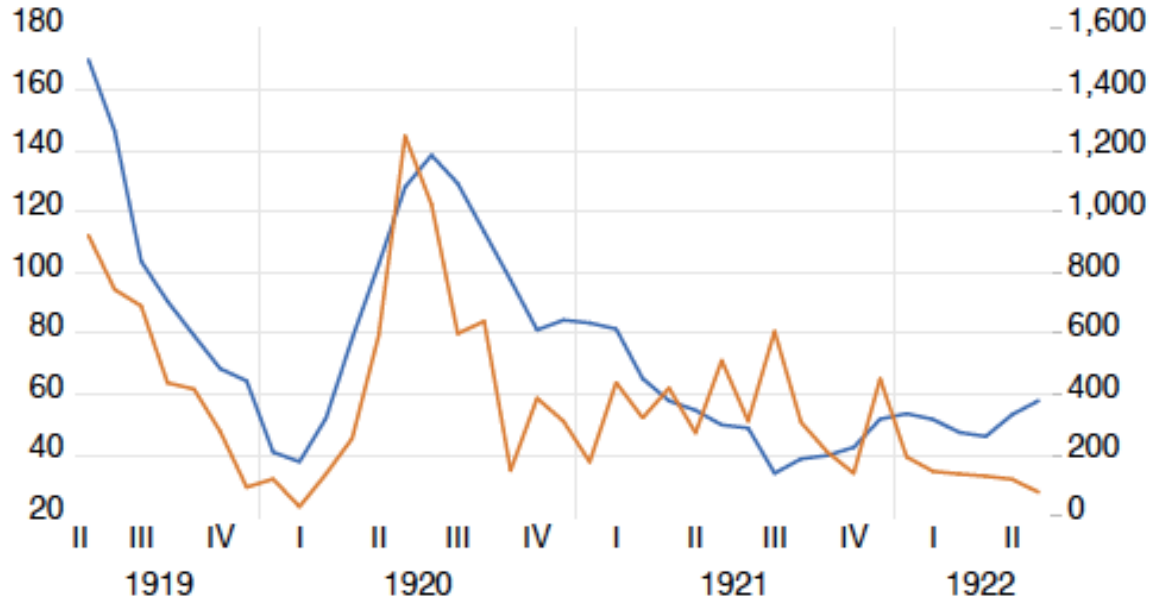
TABLE 1 - *Government Expenditure by Economic Category*
(as Percentage of Total Expenditure)

	Goods and services			Transfers and subsidies
	Current	Capital	Total	
1913	64	19	83	17
1925	52	6	58	42
1932	49	3	52	48

Source: Andic and Veverka (1963/64).

The fact that in phases of high inflation the Patinkin effect prevailed over the Olivera-Tanzi effect implies that in those of low inflation this second effect prevailed over the first, leading to a rise in the Reich's public deficit in gold marks. Paradoxically, then, as Figure 7 shows, it was in these phases that the Reich's need for seigniorage financing increased. This was mainly through an acceleration of the inflation rate rather than through an acceleration of money growth.

FIGURE 7 - *German Government Deficit and Seigniorage*



Source: Bresciani-Turroni (1937).
Legend: The blue line is seigniorage (measured on the left scale) while the red line is budget deficit (measured on the right scale).

5. Conclusions

The removal of the assumption that the government deficit is constant in real terms underlying the Cagan model and its developments, and the consideration of the Olivera-Tanzi and Patinkin effects open up new developments in the analysis of the stability of "high" and "low" price growth rates during high inflation and in the determination of the optimal inflation rate.

These developments are reflected in the German experience in the years immediately following WWI, when Germany was characterised by very high average price growth, but with alternating phases of low inflation and extremely high inflation.

In the Weimar Republic at that time, the Patinkin effect, whereby real government expenditure contract at times of high inflation, prevailed over the Olivera-Tanzi effect. This explains the fact that the Reich's deficit contracted in real terms in phases of high inflation and widened in phases of low price growth. Surprisingly, the need to cover public deficits with seigniorage increased at times of low inflation rather than at times of high inflation. In these phases, the Reichsbank increased the money supply considerably, leading to a significant increase in real terms.

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